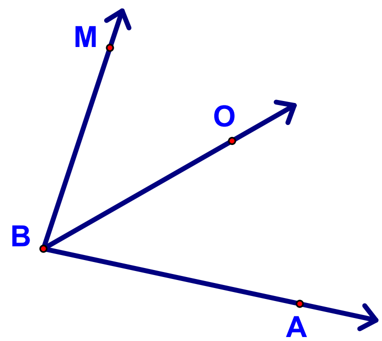
Division of Segments and Angles - Lesson 1-5

Here's your warmup!

Given: $m\angle MBA = 84°$
$m\angle ABO = 42°$

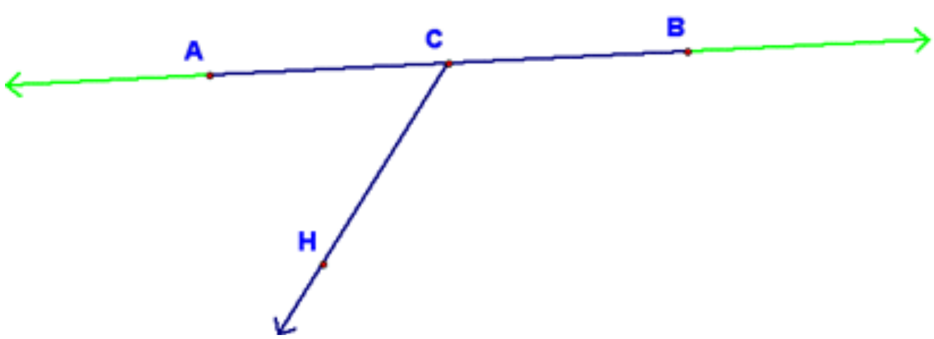
Prove: $\angle MBO \cong \angle ABO$



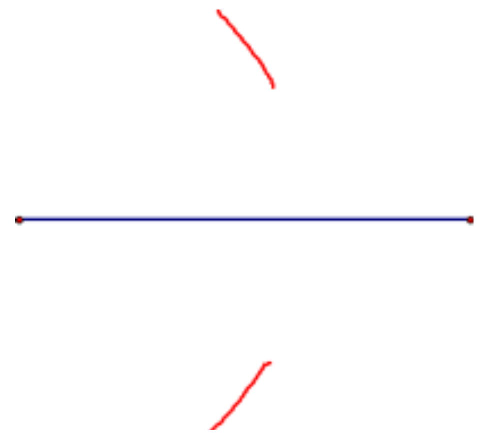
Statements	Reasons

Let's start today's lesson by defining a bisector and a midpoint.

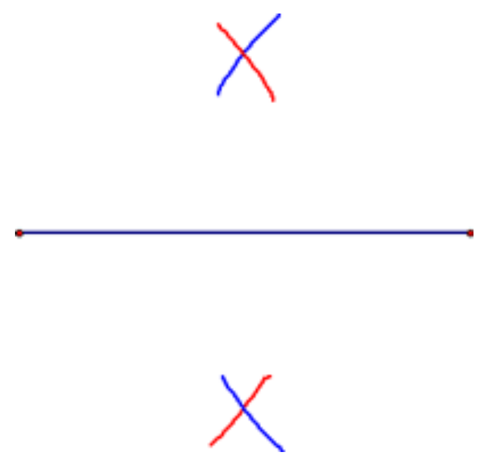
A point (or segment, ray or line) that divides a segment into two congruent segments *bisects* the segment. The bisection point is called the *midpoint* of the segment (point C in the diagram below).



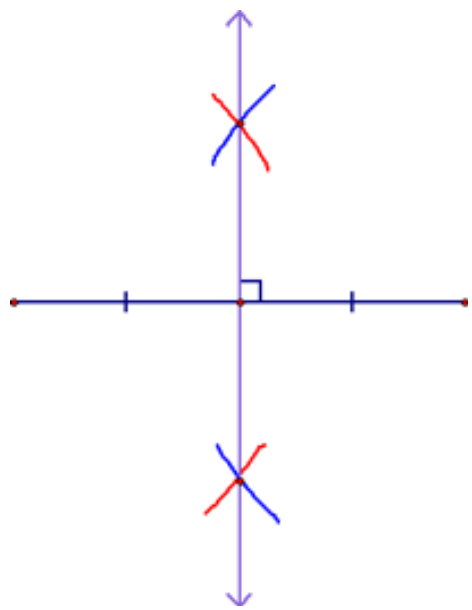
Now, get out your compass and learn how to create a perpendicular bisector of a given segment. To start, we need a segment (which you can draw using your straight edge). Then, we center our compass at one endpoint of the segment, make the radius of the compass be more than 1/2 the length of the segment (you can estimate it), and make an arc above and below the segment (see the red arcs below):



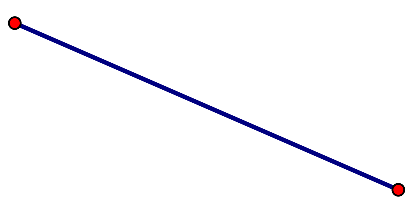
Now, with the same radius, draw similar arcs from the other endpoint:



You're almost there...just place your straight edge on the x's (X marks the spot!) and draw in the perpendicular bisector. Mark it correctly and you've done your first construction (you did it all without a protractor or a ruler!).

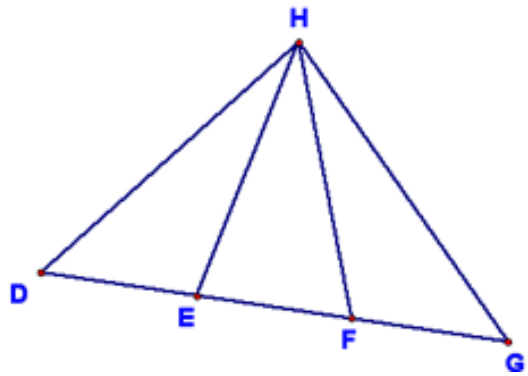


Practice this again! Here's a segment – construct it's perpendicular bisector!!



Now let's define segment trisectors and trisection points.

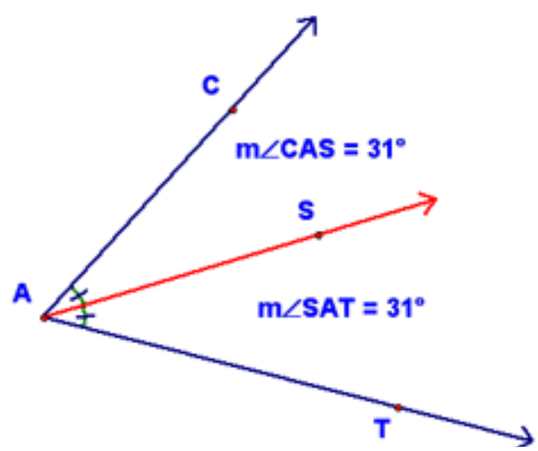
Two points (or segments, rays, or lines) that divide a segment into three congruent segments *trisect* the segment. The two points at which the segment is divided are called the *trisection points* of the segment.



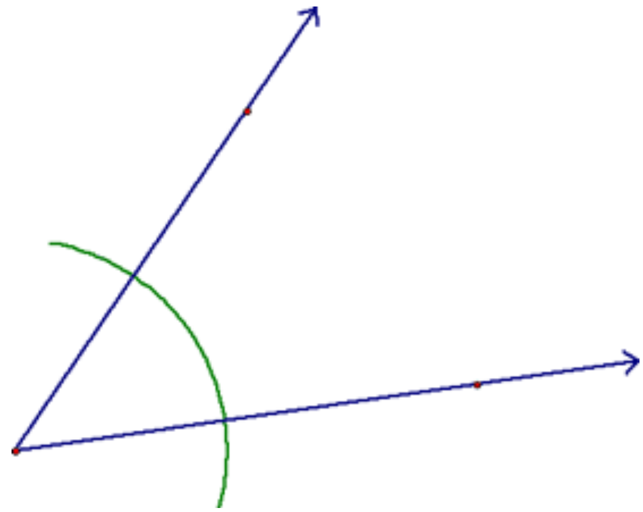
If E and F are trisection points of $\overline{DG}$, what conclusions can we draw?

There is a construction for segment trisectors (in fact, there are any number of them), but we won't cover that...if you're interested, I'm sure you can find them on the web...just Google "Segment Trisection" + "Construction." We'll now define an angle bisector.

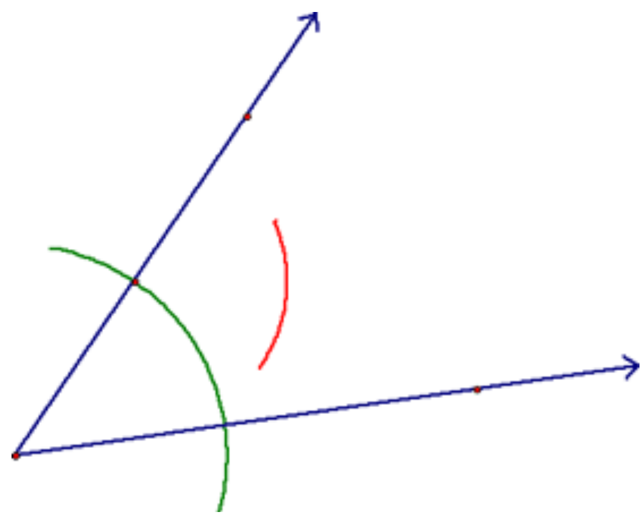
A ray that divides an angle into two congruent angles *bisects* the angle. The dividing ray is called the *bisector* of the angle.



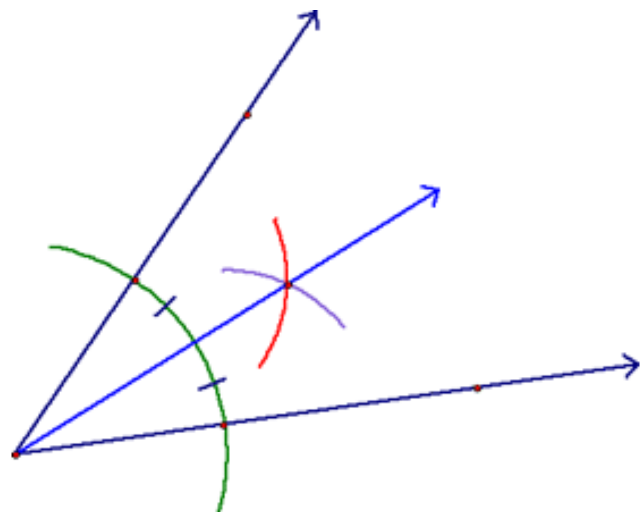
And now let's learn how to construct one! Start with an angle, center your compass at the vertex and create an arc that crosses both sides of the angle:



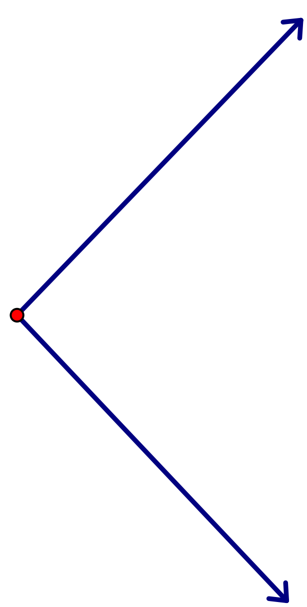
Now, center the compass at one of the points of intersection of the arc and one of the sides. Draw an arc in the center of the angle (the red arc).



Now, center the compass where the first arc intersects the other side and, with the same radius, draw another arc in the center of the angle (the purple one). Using the vertex and this "x" you can now use your straight edge to create the angle bisector. Don't forget to mark it appropriately!!

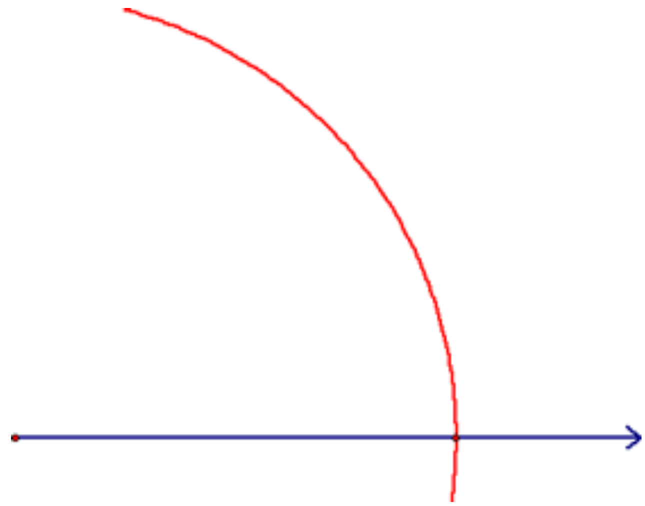


Now practice!! Here's an angle…construct it's angle bisector (don't forget to correctly label it!)…

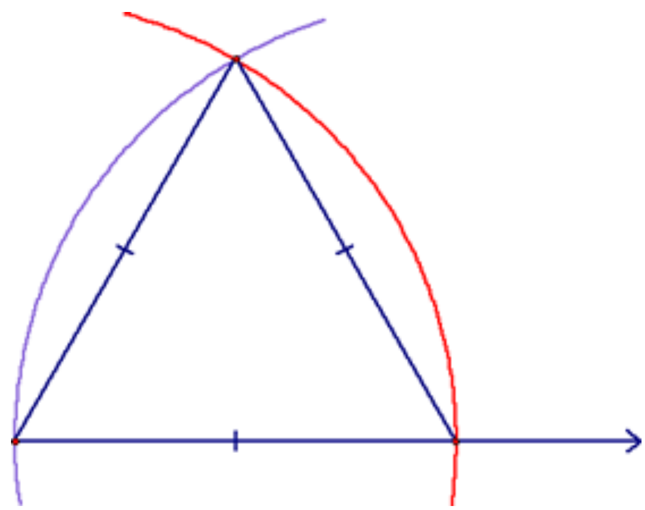


Now let's learn how to construct an equilateral triangle (in other words, a 60 degree angle). Start with a ray:

Next, center your compass at the endpoint of the ray and construct an arc like the red one below:

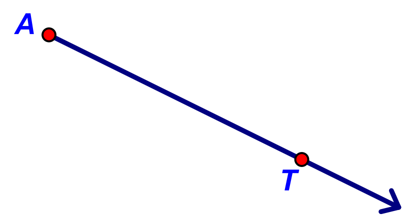


Now, center your compass at the place where the arc crosses the ray and with the same radius, construct another arc like the purple one shown below:



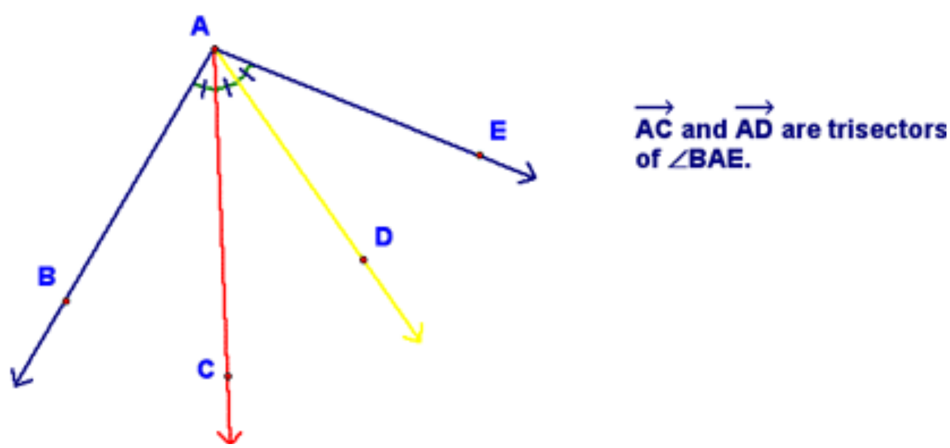
Finally, use the endpoint of the original ray and the point where "x marks the spot" to construct your equilateral triangle and/or 60 degree angle!

Practice again!! Here's a ray – construct a 60° angle that has one side as $\overrightarrow{AT}$.



Now, let's define angle trisectors:

Two rays that divide an angle into three congruent angles *trisect* the angle. The two dividing rays are called *trisectors* of the angle.



There is no construction for an angle bisector...weird, huh? Maybe you can discover one and become famous!

Finally, here are a couple of example problems...similar to what you might see on homework. Give them a try!

$\overline{EH}$ is divided by F and G in the ratio 5:3:2 from left to right. If EH = 30, find FG and name the midpoint of $\overline{EH}$.

$m\angle ABC = 35°45'$ and $\overrightarrow{BD}$ bisects $\angle ABC$.

Find $m\angle CBD$ (in degrees, minutes and seconds).