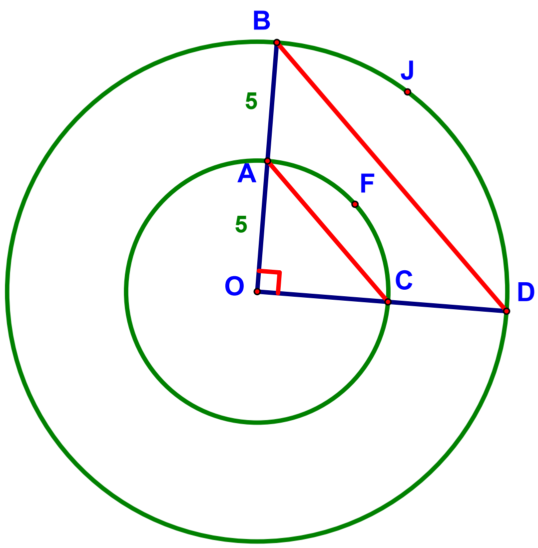
Arcs of a Circle - Lesson 10-3

Here's the warmup for today!



What fraction of the small circle is $\overset{\frown}{AFC}$?

What fraction of the large circle is $\overset{\frown}{BJD}$?

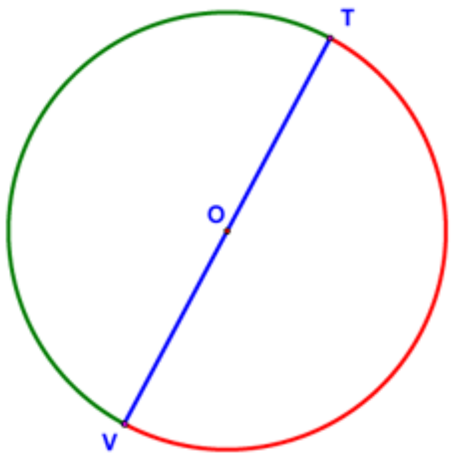
Which is longer, $\overset{\frown}{BJD}$ or $\overset{\frown}{AFC}$?

Find the lengths of $\overline{AC}$ and $\overline{BD}$.

Find the ratio $\frac{BD}{AC}$.

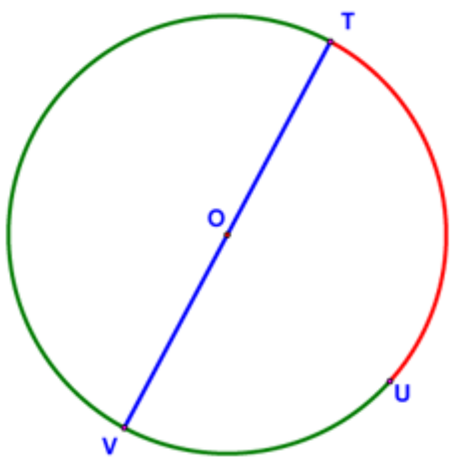
What is the ratio of the measure of $\overset{\frown}{BJD}$ to the measure of $\overset{\frown}{AFC}$?

Today we’re going to discus arcs of circles. Let’s start with a number of definitions:

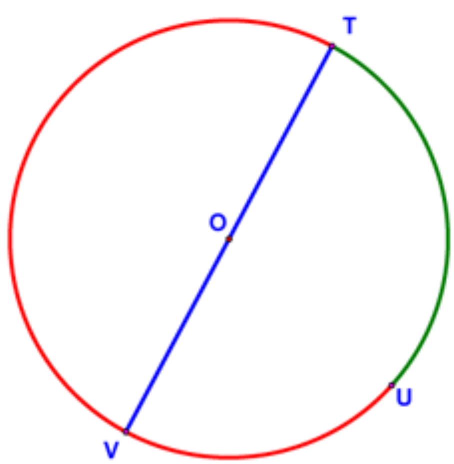


An ***arc of a circle*** is two points on the circle and the continuous (unbroken) part of the circle between the two points. The two points are called the endpoints of the arc. There is a symbol for an arc: the symbol is placed above the letters that name the endpoints of the arc. So, arc TU is written $\overset{\frown}{TU}$.

A ***semicircle*** is an arc of a circle whose endpoints are the endpoints of a diameter

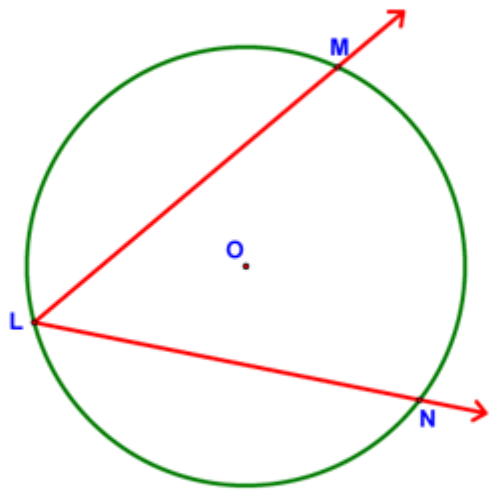


A ***minor arc*** is an arc of a circle that is smaller than a semicircle of the circle. Minor arcs are named with the letters of the two endpoints of the arc (e.g., $\overset{\frown}{TU}$).

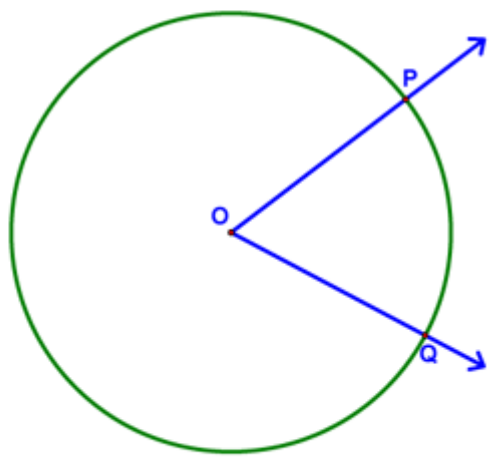


A ***major arc*** is an arc of a circle that is larger than a semicircle of the circle. Major arcs are named with the letters of three points - the first and last are the endpoints and the middle letter is any other point on the arc (e.g., $\overset{\frown}{TVU}$).

Now, we'll define a couple of angles related to circles:

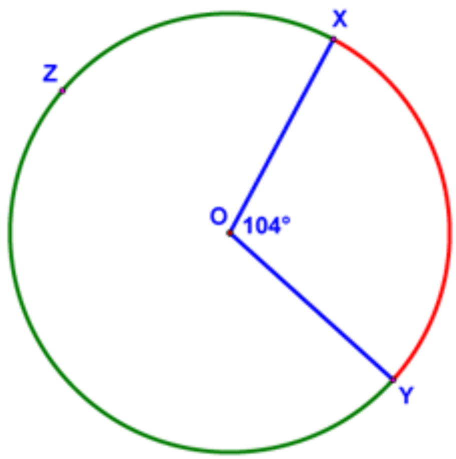


An ***inscribed angle*** is an angle whose vertex lies on a circle and whose sides contain chords of the circle (∠MLN is an inscribed angle).



A ***central angle*** is an angle whose vertex is the center of a circle and whose sides contain radii of the circle (∠POQ is a central angle).

Using these definitions, we can learn how to measure arcs and define *congruent arcs*:



The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc $\left(\text{e.g., } m\overset{\frown}{YX} = 104°\right)$.

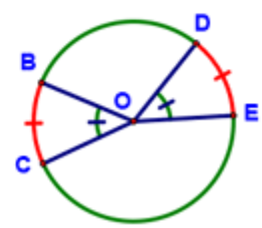
The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints $\left(\text{e.g., } m\overset{\frown}{XZY} = 256°\right)$.

Two arcs are *congruent* whenever they have the same measure and are parts of the same circle or congruent circles.

We'll conclude with the following (all of which should make sense based on the definitions above):

Theorem 78

If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.

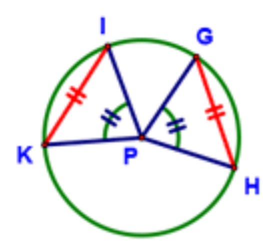


Theorem 79

If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.

Theorem 80

If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.

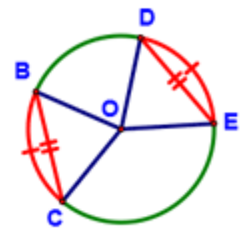


Theorem 81

If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.

Theorem 82

If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.



Theorem 83

If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.