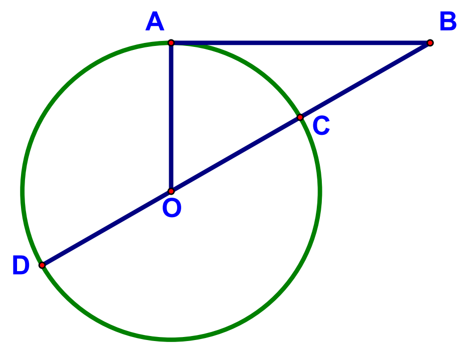
Secants and Tangents - Lesson 10-4

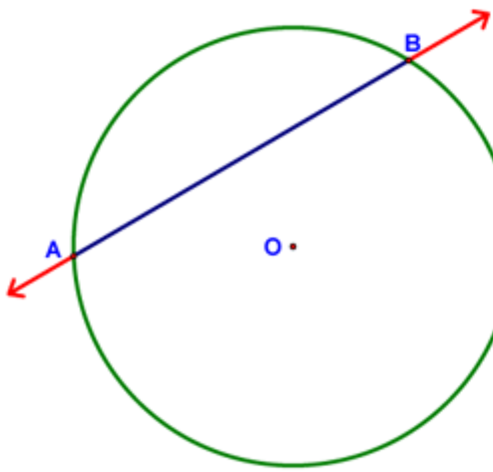
Here’s today’s warmup!

Given: ⊙O with radius 10
$\overline{AO} \perp \overline{AB}$
$m\widehat{AC} = 60^\circ$

Find: BC

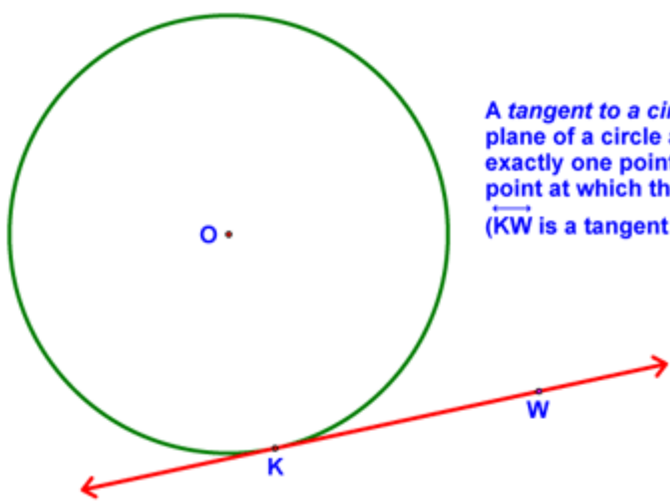


Today we'll begin by looking at the definition of *chords*, *secants*, and *tangent* of a circle:



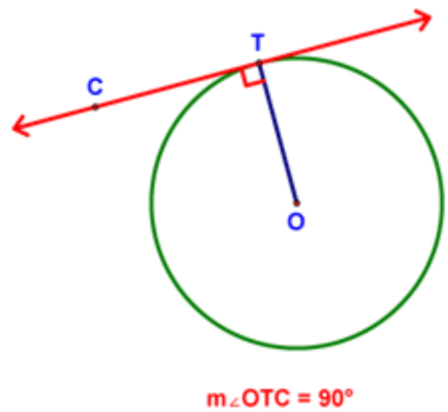
A *chord of a circle* is a segment whose endpoints lie on the circle ($\overline{AB}$ is a chord).

A *secant of a circle* is a line that contains a chord ($\overleftrightarrow{AB}$ is a secant).



A *tangent to a circle* is a line that lies in the plane of a circle and that intersects the circle at exactly one point. The *point of tangency* is the point at which the tangent touches the circle ($\overleftrightarrow{KW}$ is a tangent to $\odot O$).

Here's a property of tangents:



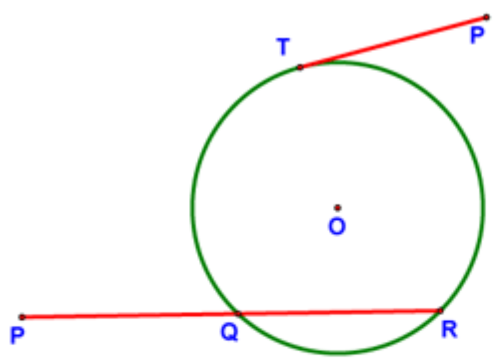
Postulate:

A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

Postulate:

If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.

Next, let's define *tangent segments* and *secant segments*:

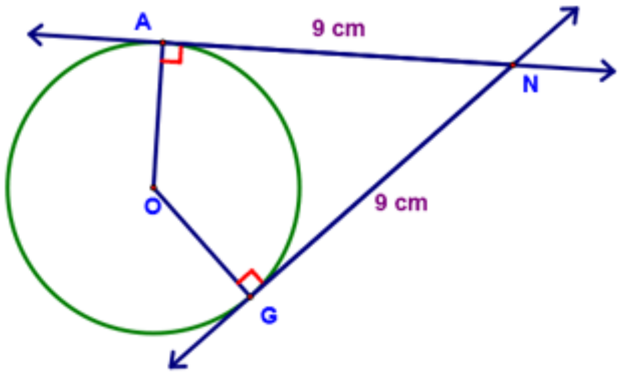


A *tangent segment* is the part of a tangent line between the point of tangency and a point outside the circle.

A *secant segment* is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

The *external part of a secant segment* is the part of a secant line that joins the outside point to the nearer intersection point.

Here is a property of tangent segments that hopefully makes sense:



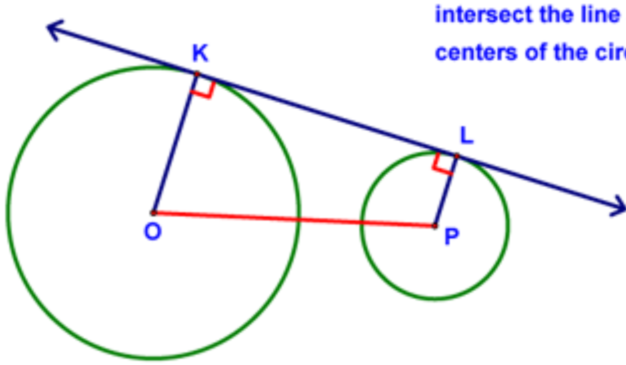
***Theorem 84*:**

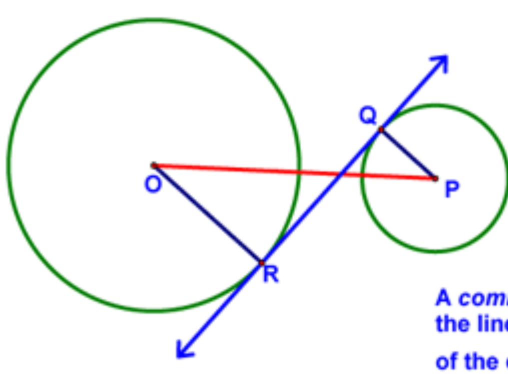
If two tangent segments are drawn to a circle from an external point, then those segments are congruent (*Two Tangents Theorem*).

And here are number of additional definitions:

A line segment that is tangent to two circles is called a *common tangent*.

A *common external tangent* does not intersect the line segment connecting the centers of the circles (e.g., $\overleftrightarrow{KL}$).

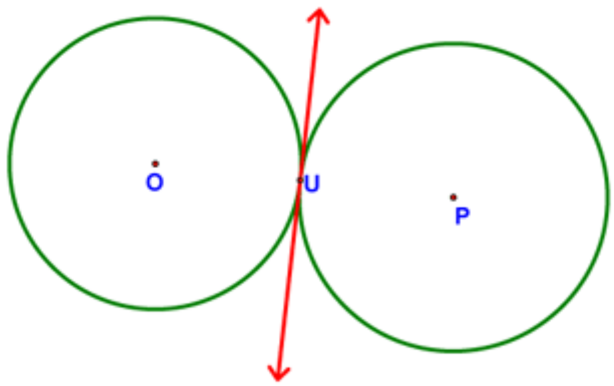




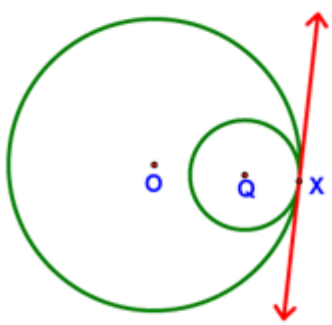
A *common internal tangent* does intersect the line segment connecting the centers of the circles (e.g., $\overleftrightarrow{RQ}$).

Tangent circles **are two circles that are tangent to the same line at the same point.**

Two circles are *externally tangent* if each of the tangent circles lies outside the other.



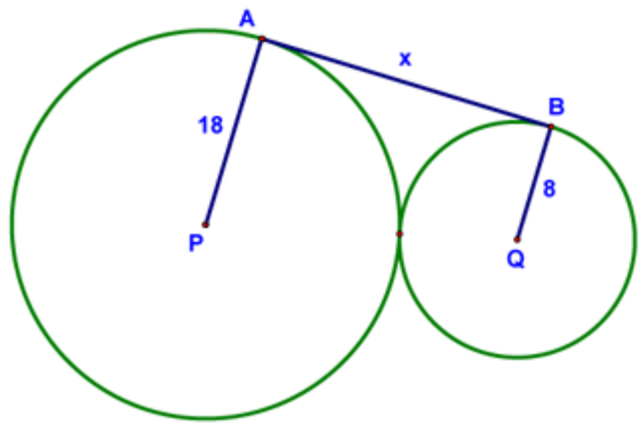
Two circles are *internally tangent* if one of the tangent circles lies inside the other.



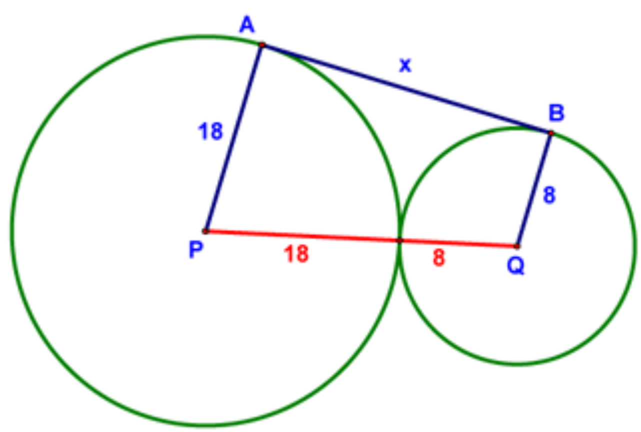
We'll finish today by talking about the Common Tangent Procedure (detailed below) and doing the following example...make sure you can follow these steps for either a common internal or common external tangent!

1. **Draw the segment between the centers.**
2. **Draw radii to the points of tangency.**
3. **Through the center of the smaller circle, draw a line parallel to the common tangent (this forms a rectangle and a right triangle.**
4. **Use the Pythagorean Theorem and properties of a rectangle to determine the length of the common tangent.**

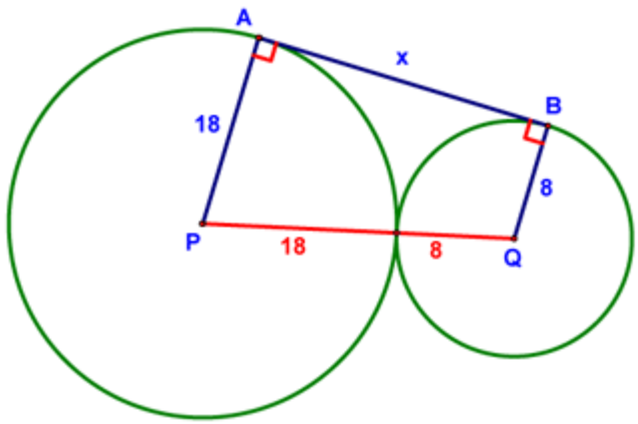
The problem was to find *x* in the diagram below:



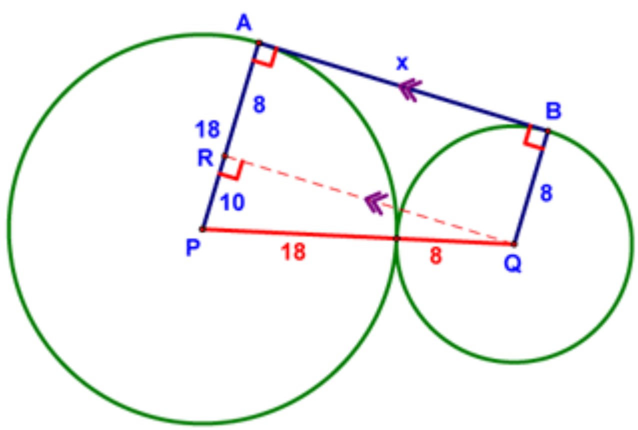
Step 1 is to draw in the segment connecting the centers of the circles (shown in red below):



Step 2 is to draw in radii to the points of tangency of the common tangent. These were already drawn for our example (segments AP and BQ), but for now, let's at least note that they form right angles with the common tangent segment.



Step 3 is to use the parallel postulate to draw in a line parallel to the tangent through the center of one of our circles (segment RQ). Note that ARQB is a rectangle, so its opposite sides are congruent. Using this fact and a little simple math, we can see that RP = 10.



Step 4, the last step, uses the Pythagorean Theorem (or a family of right triangles - in this case we have a 5-12-13 triangle), solve for the length of segment RQ, which is the same as *x*!!

AB = QR = 24 (5-12-13 Δ)

Note that this procedure (Steps 1 - 4) will also work for finding the length of a common internal tangent or common external tangent when the circles are not externally tangent to another (not touching). The picture will be different, but the result will be the same!