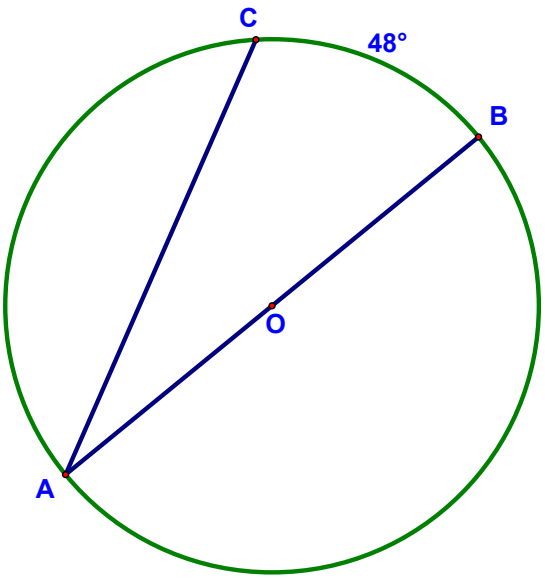
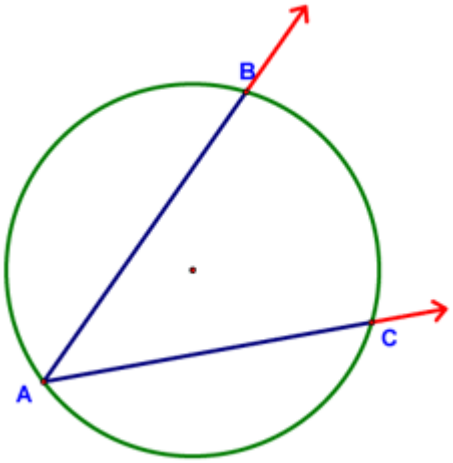
Angles Related to a Circle - Lesson 10-5

Here's the warmup!

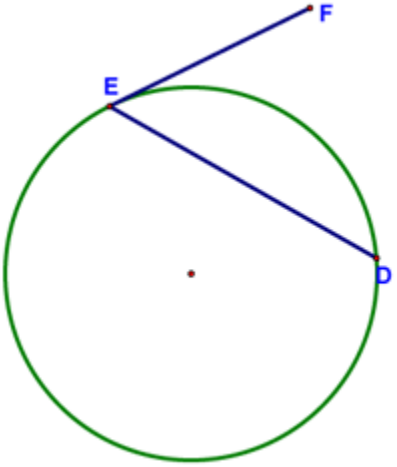
Find $m\angle A$



Today we'll start by reviewing a couple of definitions (angles with vertices ***on*** the circle):

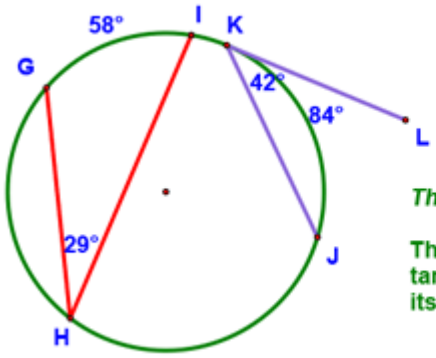


An *inscribed angle* is an angle whose vertex lies on a circle and whose sides contain chords of the circle (∠BAC is an inscribed angle).



A *tangent-chord angle* is an angle whose vertex is on a circle and whose sides are determined by a tangent and a chord that intersect at the tangent's point of tangency (∠DEF is a tangent-chord angle).

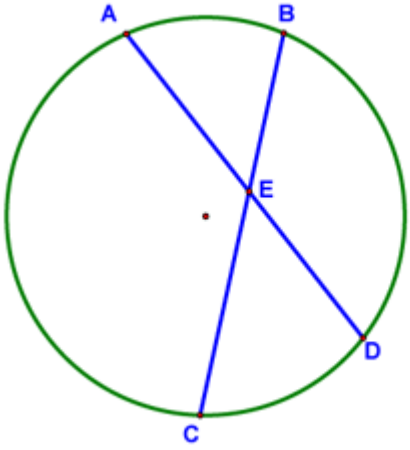
Here's a property of these types of angles:



Theorem 85

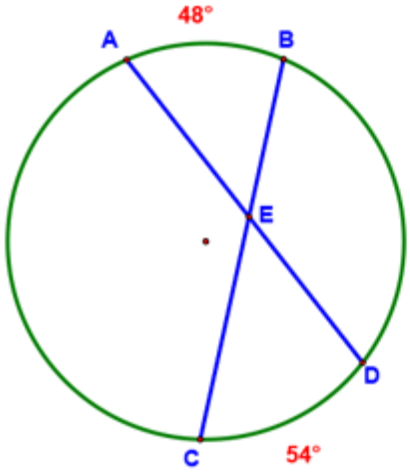
The measure of an inscribed angle or a tangent-chord angle is one-half the measure of its intercepted arc.

Next, we will define *chord-chord angles* (angles with vertices ***inside*** the circle):



A *chord-chord angle* is an angle formed by two chords that intersect inside a circle but not at the center (∠AEB, ∠CED, ∠AEC, ∠BED are chord-chord angles).

And learn a property about them:

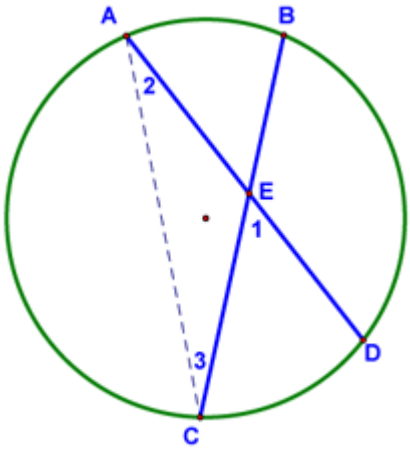


Theorem 86

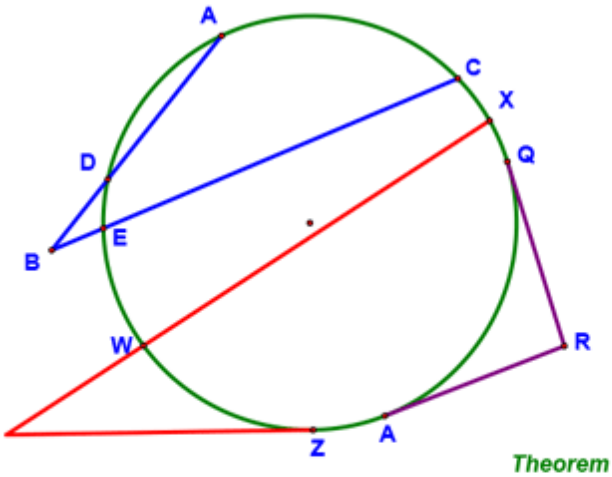
The measure of a chord-chord angle is one-half the sum of the measures of the arcs intercepted by the chord-chord angle and its vertical angle.

e.g., $m\angle AEB = \frac{1}{2}(48 + 54) = 51°$

Here's a hint for how to prove Theorem 86...see if you can do it!



Now, let's define three more types of angles related to circles (those with their vertices ***outside*** of the circle and learned a property about them:



A *secant-secant angle* is an angle whose vertex is outside a circle and whose sides are determined by two secants.

A *secant-tangent angle* is an angle whose vertex is outside a circle and whose sides are determined by a secant and a tangent.

A *tangent-tangent angle* is an angle whose vertex is outside the circle and whose sides are determined by two tangents.

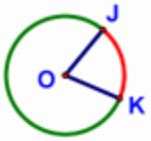
Theorem 87

The measure of a secant-secant angle, a secant-tangent angle, or a tangent-tangent angle (vertex outside the circle) is one-half the difference of the measures of the intercepted arcs.

Here's a summary of all those theorems. This page is really important so make sure you have it down!!

Angle-Arc Summary

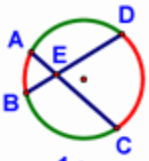
Central Angle



$m\angle KOJ = m\overset{\frown}{JK}$

Vertex at center ⇒ equal

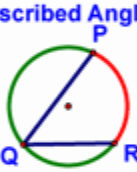
Chord-Chord Angle



$m\angle DEC = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$

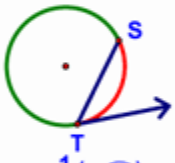
Vertex inside ⇒ half the sum

Inscribed Angle



$m\angle PQR = \frac{1}{2}\left(m\overset{\frown}{PR}\right)$

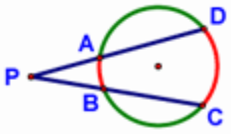
Tangent-Chord Angle



$m\angle T = \frac{1}{2}\left(m\overset{\frown}{ST}\right)$

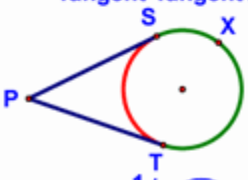
Vertex on circle ⇒ half the arc

Secant-Secant Angle



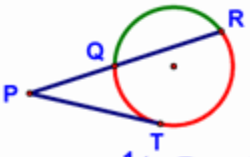
$m\angle P = \frac{1}{2}\left(m\overset{\frown}{CD} - m\overset{\frown}{AB}\right)$

Tangent-Tangent Angle



$m\angle P = \frac{1}{2}\left(m\overset{\frown}{SXT} - m\overset{\frown}{ST}\right)$

Secant-Tangent Angle



$m\angle P = \frac{1}{2}\left(m\overset{\frown}{RT} - m\overset{\frown}{QT}\right)$

Vertex outside circle ⇒ half the difference

Let's finish by doing the following example: