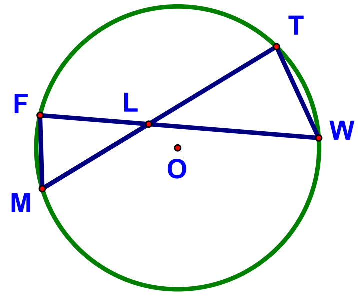
The Power Theorems - Lesson 10-8

Here's a warmup!

Given: F, M, T, W are on ⊙O

Prove: $\triangle FLM \sim \triangle TLW$

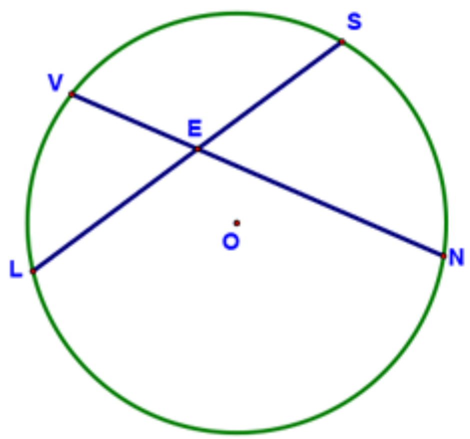


Statements	Reasons

Today we're going to learn a number of cool theorems (called the Power Theorems...sounds cool, huh?) that are based on similar triangles. The first is shown below and should be pretty straight forward to understand if you look at proving two triangles similar (which, of course implies that corresponding sides are proportional):

Theorem 94

If two chords of a circle intersect inside the circle, then the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other chord (Chord-Chord Power Theorem).



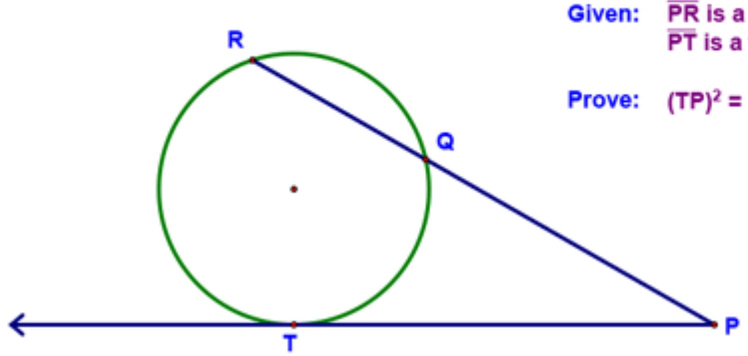
VE · EN = LE · ES

Why??

The second power theorem is show below:

Theorem 95

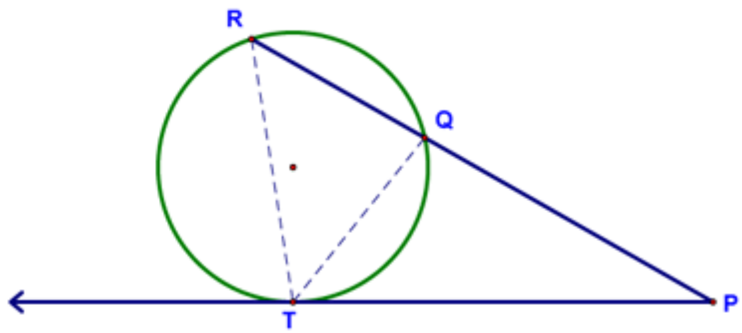
If a tangent segment and a secant segment are drawn from an external point to a circle, then the square of the measure of the tangent segment is equal to the product of the measures of the entire secant segment and its external part (Tangent-Secant Power Theorem).



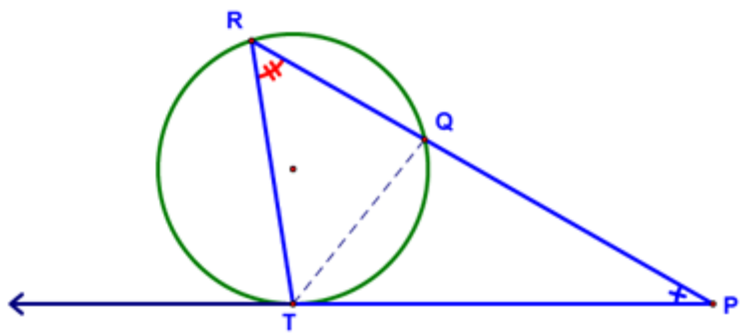
Given: $\overline{PR}$ is a secant segment
$\overline{PT}$ is a tangent segment

Prove: $(TP)^2 = (PQ)(PR)$

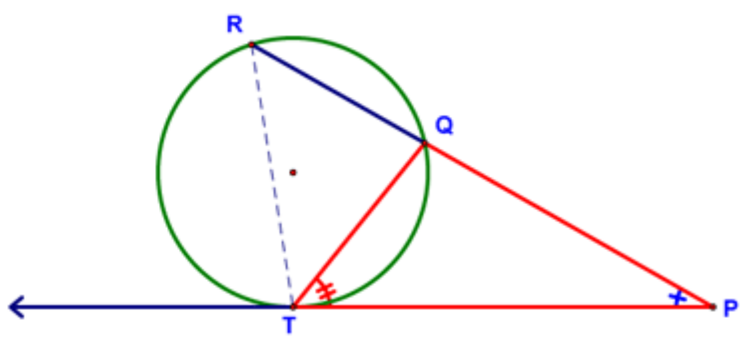
Proving the Tangent-Secant Power Theorem is a little more difficult, but still involves similar triangles. The key is to draw in the two auxiliary lines shown below ($\overline{RT}$ & $\overline{QT}$):



Note that ∠R and ∠QTP are congruent (why?) and that ∠P is in both △RTP and △QTP. With this information, you should see that the triangle below (△RTP)



and △TQP shown below are similar.

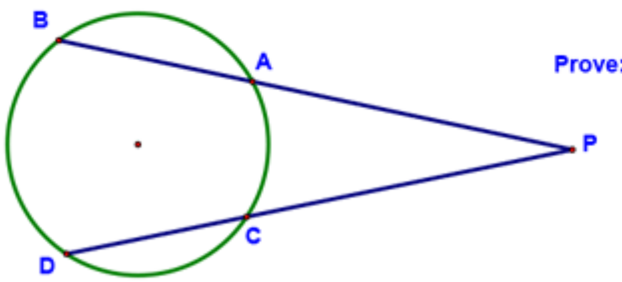


Since $\triangle RTP \sim \triangle TQP$, this implies that $\frac{TP}{PQ} = \frac{PR}{TP}$ or $(TP)^2 = (PQ)(PR)$. Woohoo!

The last power theorem (the Secant-Secant Power Theorem) is shown below:

Theorem 96

If two secant segments are drawn from an external point to a circle, then the product of the measures of one secant segment and its external part is equal to the product of the measures of the other secant segmnet and its external part (Secant-Secant Power Theorem).



Given: $\overline{PB}$ and $\overline{PD}$ are secant segments

Prove: (PA)(PB) = (PC)(PD)

This theorem is proved using the two triangles shown in the diagram below. You should be able to figure this one out!

Now, please try the following example problem:

Find x in the diagram below: