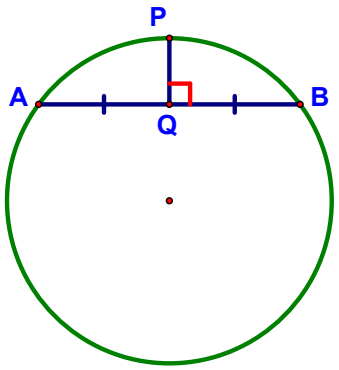
Circumference and Arc Length - Lesson 10-9

Here's the warmup!

Given: **AB = 20**
PQ = 2

Find: **The diameter of the circle.**

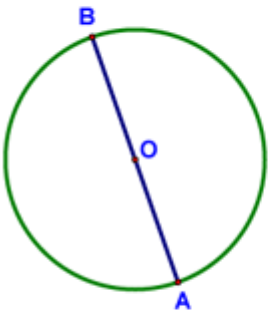


Let's start today by reviewing what we know about the definition of circumference of a circle and the relationship between the circumference and the diameter of any circle:

The *circumference* of a circle is its perimeter.

Postulate

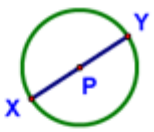
$C = \pi d$



Diameter of Circle O = 7.029 cm

Circumference ⊙OB = 22.082 cm

$$\frac{\text{Circumference } \odot OB}{\text{Diameter of Circle O}} = 3.14159$$



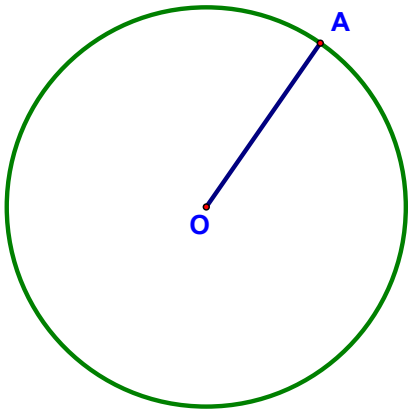
Diameter of Circle P = 3.112 cm

Circumference ⊙PY = 9.776 cm

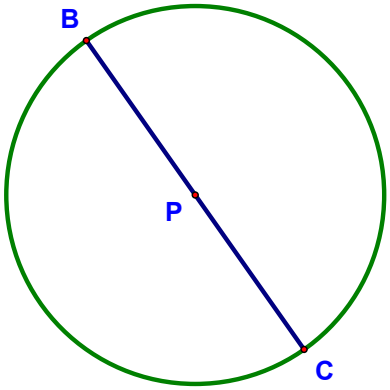
$$\frac{\text{Circumference } \odot PY}{\text{Diameter of Circle P}} = 3.14159$$

That number π is a pretty important one!!

Let's try a couple of examples of using this knowledge:



The circumference of ⊙*O* is 12π meters. What is AO?

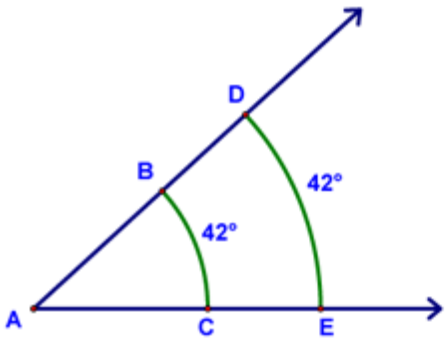


BC = 3 meters. What is the circumference of ⊙P?

Next, let's define *arc length*:

The *length of an arc* (arc length) is some fraction of the circumference of the circle.

Let's compare this to arc measure. Note that two arcs can have the same measure, but different lengths.

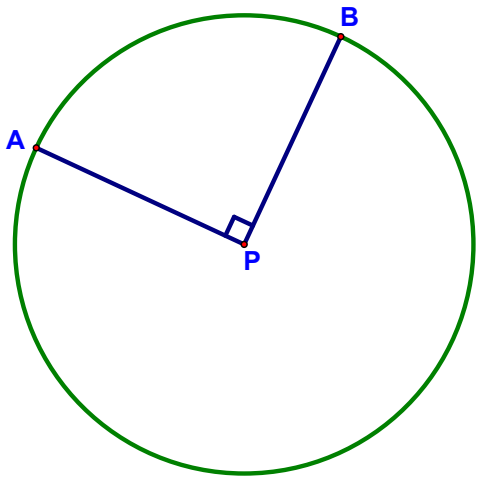


Arc measure **(as opposed to arc length) is some fraction of 360°. *Arc length* is measured, like other lengths, in some unit of measure (e.g., cm or in).**

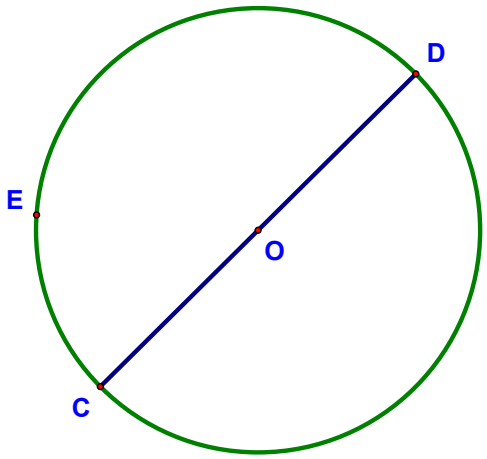
$\overset{\frown}{BC}$ and $\overset{\frown}{DE}$ have the same *measure*, but clearly have different *lengths*.

Now, try the following problems:

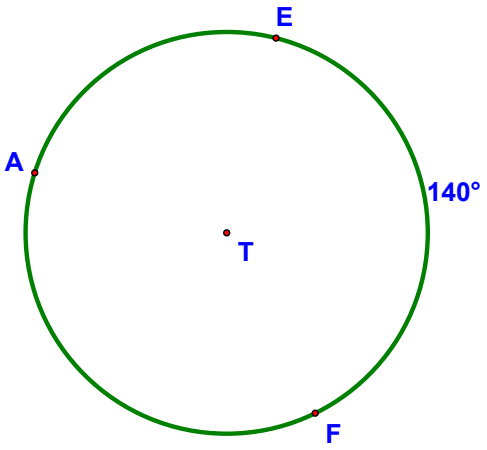
What is the length of $\overset{\frown}{AB}$? The radius of ⊙P is 12 meters.



What is the arc length of $\overset{\frown}{CED}$? The diameter of ⊙O is 8 in.



What is the arc length of $\overset{\frown}{EF}$? The radius of ⊙T is 36 feet.



We can summarize this work with the following theorem:

Theorem 97

The length of an arc is equal to the circumference of its circle times the fractional part of the circle determined by the arc.

$$\text{Length of } \overset{\frown}{PQ} = \left(\frac{m\overset{\frown}{PQ}}{360}\right)\pi d$$

where d is the diameter and $\overset{\frown}{PQ}$ is measured in degrees

Here's a final application of this theorem – see if you can figure it out!

Find the measure of the diameter of a circle that has an arc that has both measure 80° and length 88π cm.