Areas of Regular Polygons - Lesson 11-5

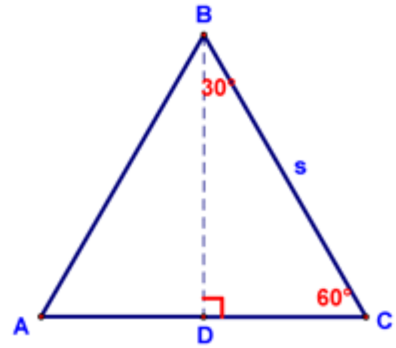
Here's the warmup!

a. **Find the area of an equilateral triangle with sides of length 8**

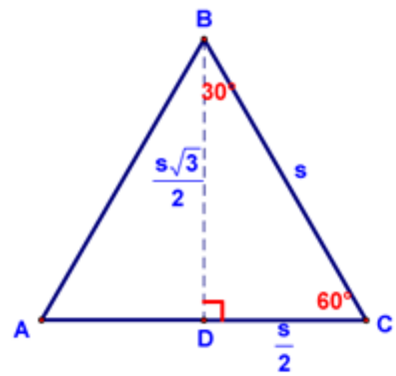
b. **Find the area of a regular hexagon with sides of length 8**

c. **Find the ratio of the area of the hexagon to the triangle.**

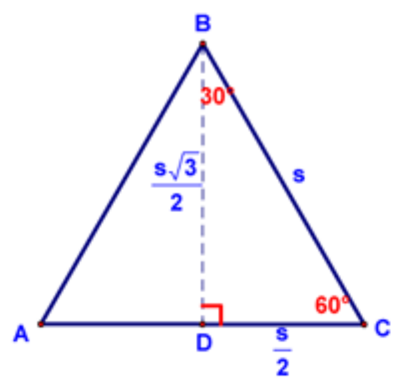
Today we're going to start by deriving a formula for the area of an equilateral (regular) triangle. If you start with an equilateral triangle with side length s:



And then do the standard procedure for finding the length of the altitude (love those 30-60-90 triangles!),



You are then able to use the formula for the area of a triangle to derive a new formula,



$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(s)\left(\frac{s\sqrt{3}}{2}\right)$$

$$A = \frac{s^2\sqrt{3}}{4}$$

which can be summarized as follows:

Theorem 104:

The area of an equilateral triangle equals the product of one-fourth the square of a side and the square root of 3.

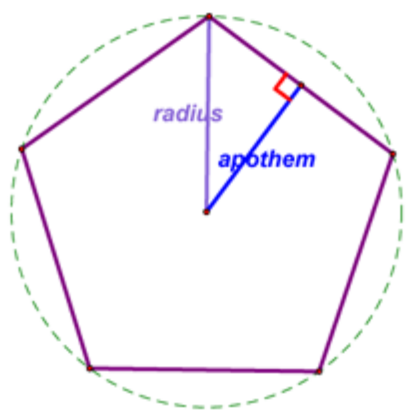
$$A = \frac{s^2\sqrt{3}}{4}$$

where s is the length of a side.

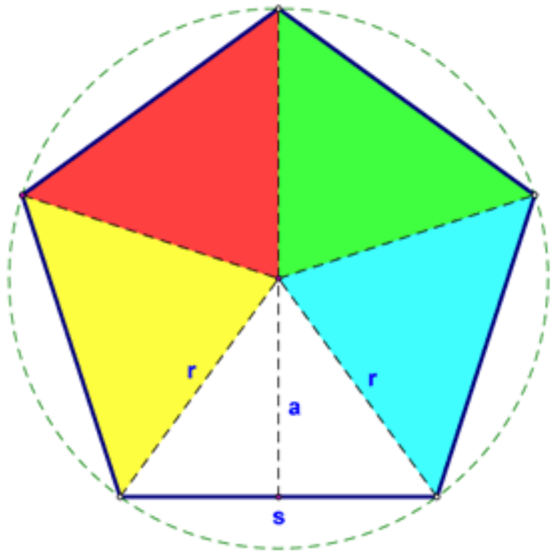
Next, we can generalize an area formula for all regular polygons. To start, we need to define the radius and the apothem of a regular polygon:

The *apothem of a regular polygon* is a perpendicular segment from the center of the polygon's circumscribed circle to a side of the polygon (joins the center to the midpoint of a side).

The *radius of a regular polygon* is a segment joining the center of the polygon's circumscribed circle to any vertex.

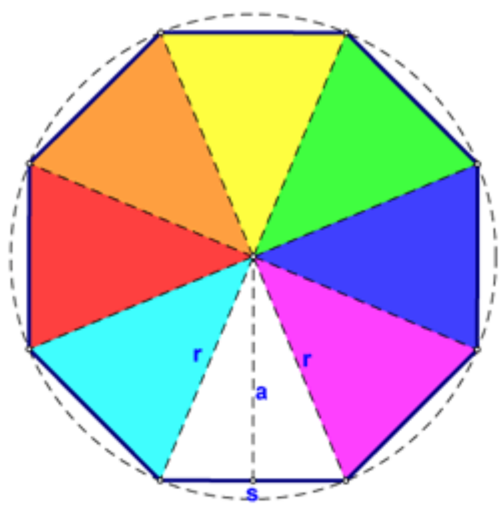


If we look at a pentagon, you should be able to see how the formula for its area would be as shown in the table below (you should fill in the box below the 5):



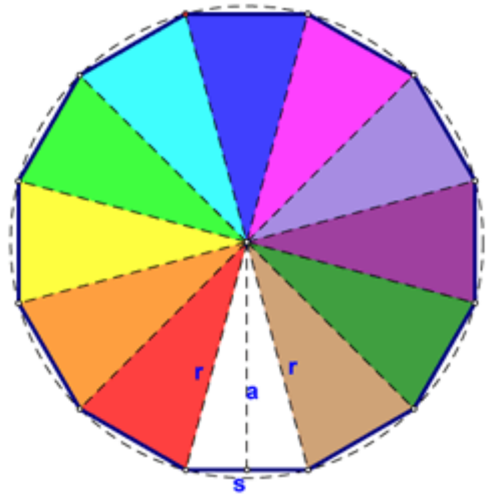
# of sides	5	8	12	···	*n*
Area of reg. polygon				···	

The same can be done for an octagon (now fill in the box below the 8):



# of sides	5	8	12	···	*n*
Area of reg. polygon				···	

and a dodecagon (yes…fill in the box below the 10).



# of sides	5	8	12	…	*n*
Area of reg. polygon				…	

In conclusion, we can derive the general formula to find the area of any given regular polygon:

# of sides	5	8	12	…	*n*
Area of reg. polygon				…	

Theorem 105

The area of a regular polygon is given by the formula $A=\frac{1}{2}asn$, where A is the area, a is the apothem, s is the length of each side, and n is the number of sides of the regular polygon.

Because the length of each side times the number of sides is the perimeter, $sn=p$, where p is the perimeter. Therefore, the formula for the area of a regular polygon can also be written $A=\frac{1}{2}ap$.