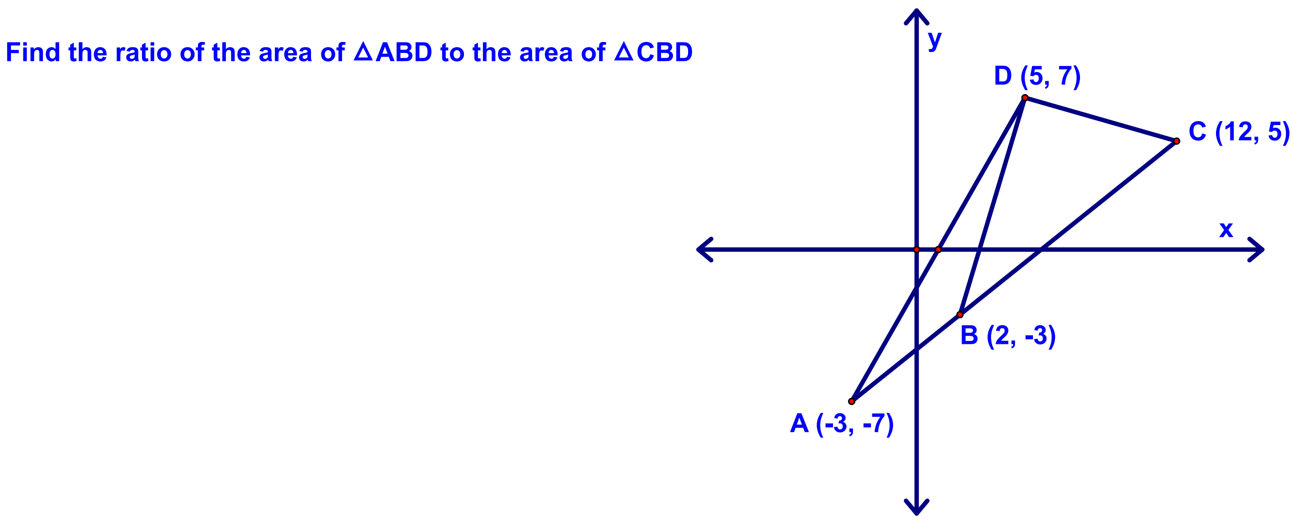
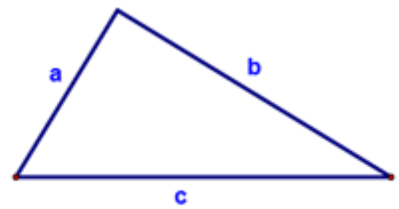
Hero's and Brahmagupta's Theorems - Lesson 11-8

Here's the warmup for today:



Today we're going to learn two formulas that were developed 2000-3000 years ago. The first was discovered by Hero of Alexandria and is therefore called Hero's Formula. It helps us to find the area of a triangle given the lengths of its three sides:



Theorem 109 (*Hero's Formula*)

$$A_{\triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

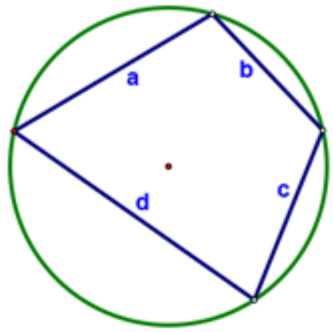
where a, b, and c are the lengths of the sides of the triangle and $s = \text{semiperimeter} = \dfrac{a+b+c}{2}$.

An example of using this to find the area of a triangle follows:

Find the area of a triangle with sides 3, 6, and 7.

Simply apply the formula!

The second formula was developed by a Hindu mathematician named Brahmagupta. His formula allows us to find the area of cyclic quadrilaterals:



Theorem 110 (*Brahmagupta's Formula*)

$$A_{\text{Cyclic Quadrilateral}} = \sqrt{(s - a)(s - b)(s - c)(s - d)}$$

where a, b, c, and d are the lengths of the sides of the quadrilateral and $s = \text{semiperimeter} = \dfrac{a + b + c + d}{2}$.

Here's an example problem:

Find the area of the inscribed quadrilateral with sides 2, 7, 6, and 9.

Again, start by finding the semiperimeter and then just use the formula: