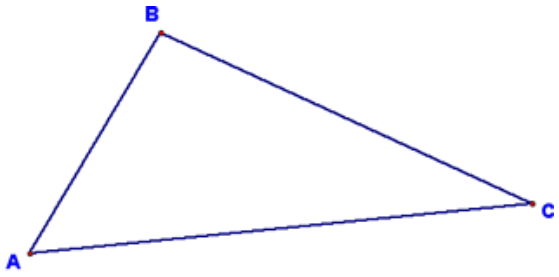
Three Ways to Prove Triangles Congruent - Lesson 3-2

Here's the warmup!

Using your rulers and a protractor, draw a triangle with the following specifications:

a. One side is 6 cm.
b. One side is 8 cm.
c. One angle is 40°.

Today, we're going to start by making sure that we understand what an included angle or included side is:



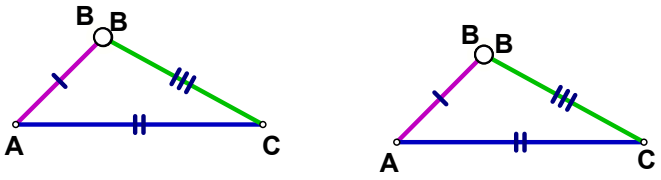
In the diagram above, $\angle A$ is included by sides $\overline{AB}$ and $\overline{AC}$.

Side $\overline{BC}$ is included by $\angle B$ and $\angle C$.

Now in a triangle, there are 3 sides and 3 angles, so this implies that to prove that two triangles are congruent, we'd have to prove that all three corresponding sides and all three corresponding angles were congruent. However, we will soon discover that there are some shortcuts for proving congruence. The six possibilities (remember that we eliminated the possibilities that included only one part of corresponding triangles congruent and two parts of corresponding triangles congruent), which we could use are:

1. SSS (Side, Side, Side)
2. SSA (Side, Side, Angle)
3. SAS (Side, Angle, Side)
4. ASA (Angle, Side, Angle)
5. SAA (Side, Angle, Angle)
6. AAA (Angle, Angle, Angle)

Given this, we should look at these possibilities, starting with SSS, which is a shortcut for proving triangle congruence since, when given three fixed sides, we can only create congruent triangles.



What can you say about triangles whose corresponding three sides are congruent?

Givens:

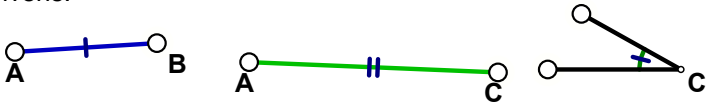
If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent (*SSS Congruence Postulate*).

Next, let's consider the SSA case. This one doesn't work (see below). Remember the difference between SSA and SAS. In the latter, the angle is *included* between the two sides, whereas in the former, the angle is *not included* between the two sides.

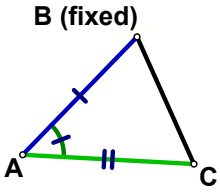
Try to connect the points labeled B in the broken triangles. Can you create two non-congruent triangles?

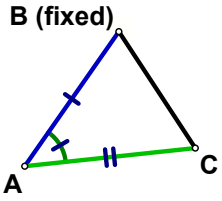
How many triangles can be formed given two sides and the angle not between them?

Givens:



Next, we looked at SAS, which also helps us to prove triangle congruence:



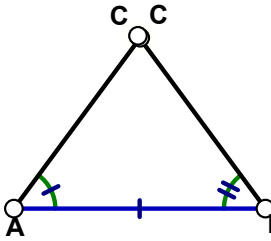


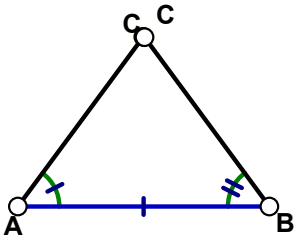
Try to connect the points labeled B in the broken triangles. Can you make two triangles that aren't congruent?

If two sides and the included angle of one triangle are congruent to the corresponding two sides and included angle in another triangle, then the triangles are congruent (*SAS Congruence Postulate*).

Givens:

Now let's look at ASA:

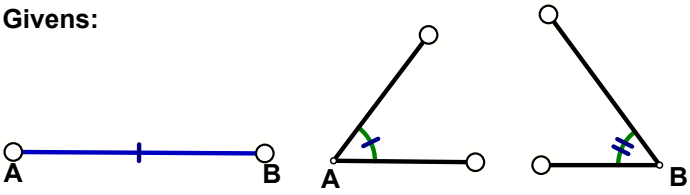




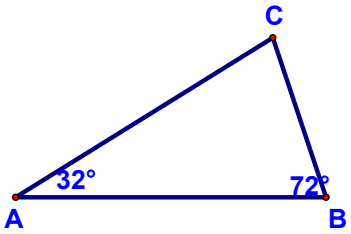
Try to connect the points labeled C in the broken triangles. Can you make triangles that aren't congruent?

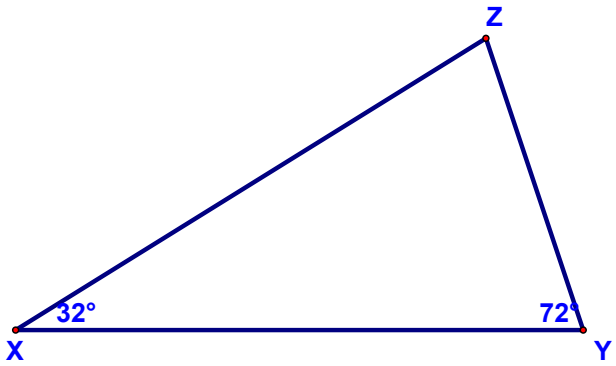
If two angles and the included of one triangle are congruent to the corresponding two angles and included side of another triangle, then the triangles are congruent (*ASA Congruence Postulate*).

Givens:



We should also at the AAA case, which we had already agreed didn't work (remember we talked about it when considering AA, which is really AAA?)



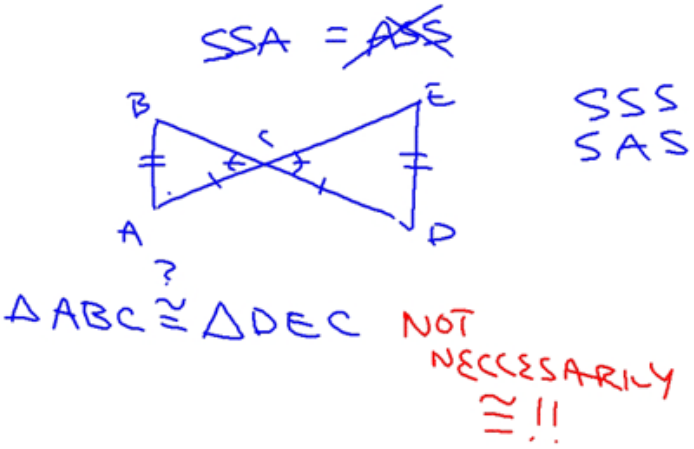


Is △ABC ≅ △XYZ?

Not necessarily (they can be congruent if AB=XY).

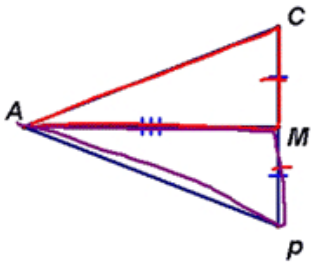
Therefore knowing that two angles in corresponding triangles are congruent is not enough information to show that the triangles are congruent.

Here are some examples. You should be able to see why these either do or do not show enough information to prove congruent triangles:



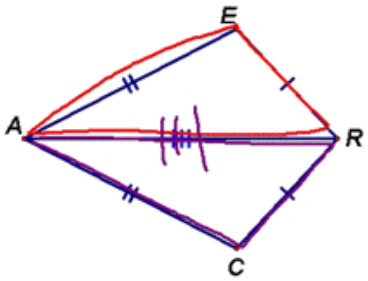
$\overline{AM}$ is a median.

$\Delta CAM \cong \Delta$ CBD, since we only know two parts are congruent.



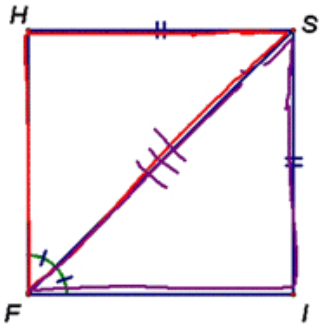
$\Delta ARE \cong \Delta ARC$

Why? SSS

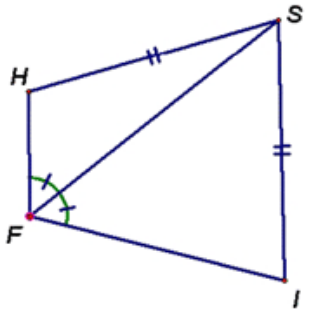


$\Delta FSH \cong \Delta$ CBD

Why? We only have SSA.

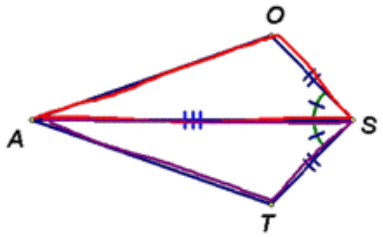


You should be able to why this is true by looking at the following diagram, which has the same limitations, but clearly shows two non-congruent triangles:



$\triangle SAT \cong \triangle SAO$

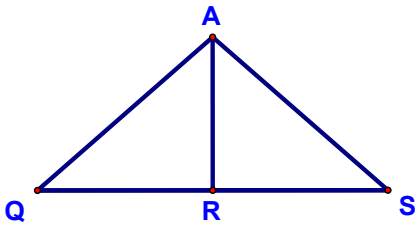
Why? SAS



Finally, let's go through a couple of example proofs...see how you do in terms of getting the correct steps!! For those who stress out about the number of steps, I did the first one in 5 steps and the second in 10!

Given: $\overline{AQ} \cong \overline{AS}$
R is the midpoint of $\overline{QS}$.

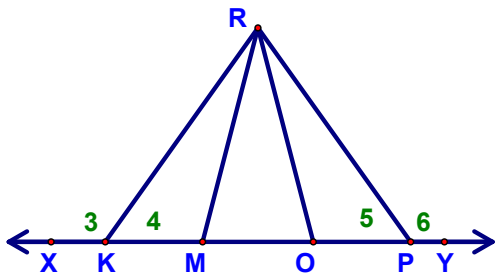
Prove: $\triangle AQR \cong \triangle ASR$



Statements	Reasons

Given: $\angle 3 \cong \angle 6$
$\overline{KR} \cong \overline{RP}$
$\angle KRO \cong \angle PRM$

Prove: $\triangle KRM \cong \triangle PRO$



Statements	Reasons