# Mr. Baroody's Web Page

Detours and Midpoints - Lesson 4-1

Today, we are going to formally go through an explanation of something we've been doing for a while. It's called "proofs with detours." You should be getting pretty comfortable with these by now...you have to prove one (or more) sets of triangles congruent in order to get enough information to prove other triangles (or parts thereof) congruent. Here is our warmup, which is an example of a detour proof:

Given: $\overline{CB} \cong \overline{CD}$
$\overline{CG} \cong \overline{FC}$
$\angle AGB \cong \angle EFD$

**Prove:** $\overline{GA} \cong \overline{FE}$
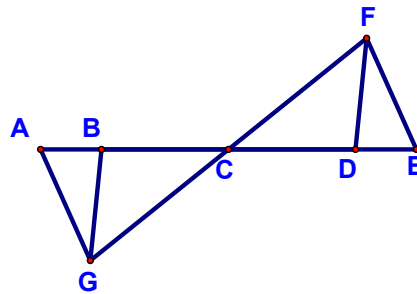
Statements	Reasons

There will be plenty of other examples in today's classwork...make sure you understand all of these! The real key is that after you have more than one pair of triangles congruent, you need to indicate from which pair you're CPCTCing by putting the appropriate step number in parentheses after the CPCTC. Also – the more detours, the more different ways proofs can be solved!

Next, let's talk about finding the midpoint of a segment given it's endpoints. This should be pretty straight forward for you – find the coordinates of the midpoints of each of the segments below.
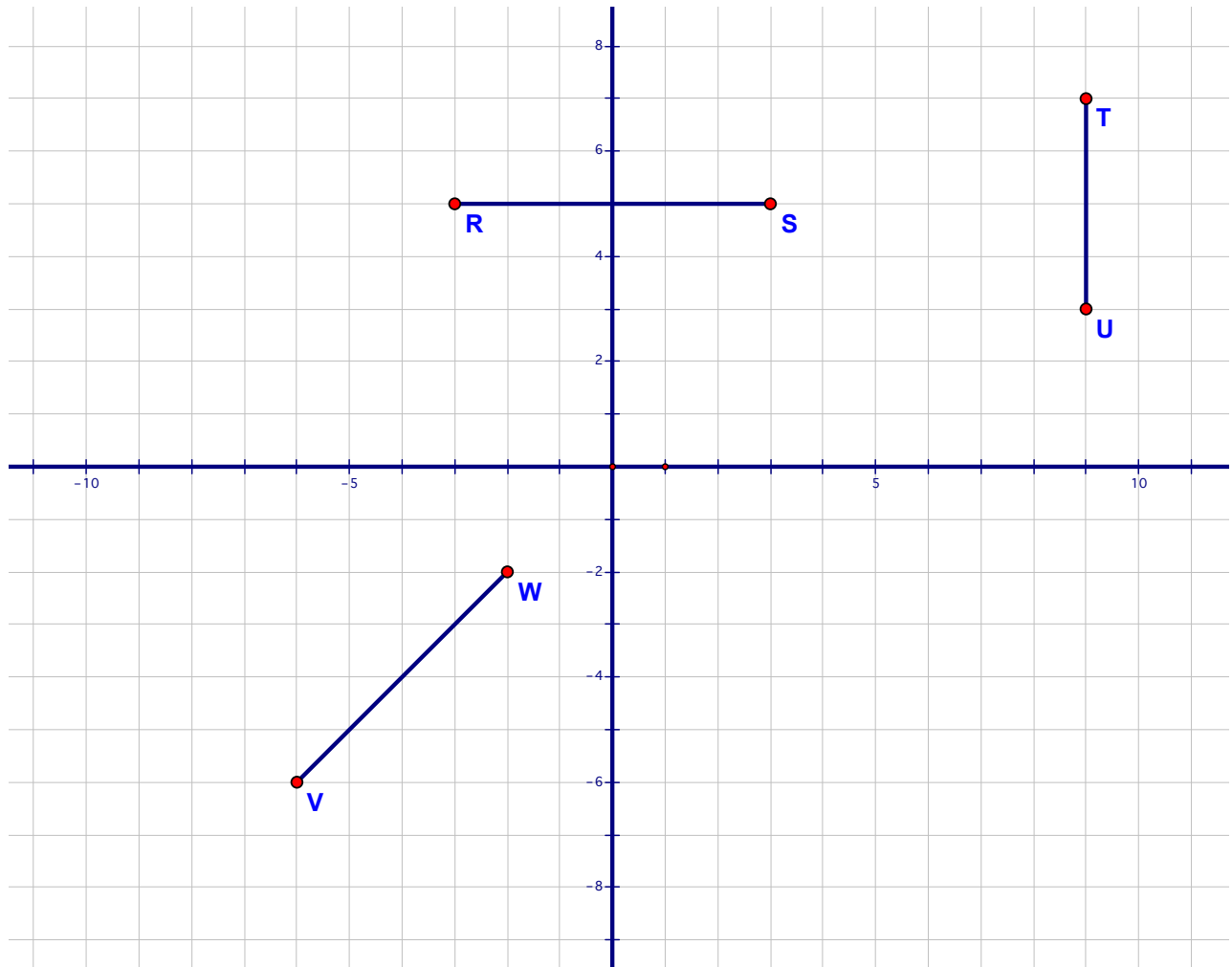
You should come to the conclusion that finding the midpoint of a line segment given it's endpoint's coordinates, was as easy as finding the *average of the x values and the average of the y values* (see the diagram and associated theorem on the next page).
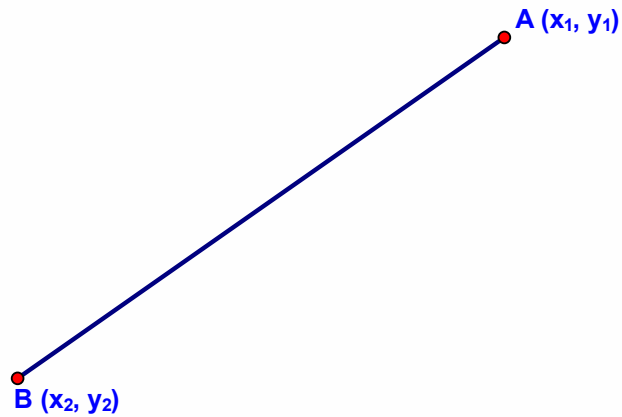
A (x₁, y₁)

B (x₂, y₂)

Theorem 23 - (Midpoint Formula) - If A = (x_1, y_1) and B = (x_2, y_2) are the coordinates of the endpoints of a segment, then the coordinates of the midpoint,

$$M = (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

As an example of using this formula, see if you can solve the following problem:

Find the coordinates of the point at which the median from A intersects $\overline{BC}$.

C (6, 10)

M

A (14, 5)

B (2,4)
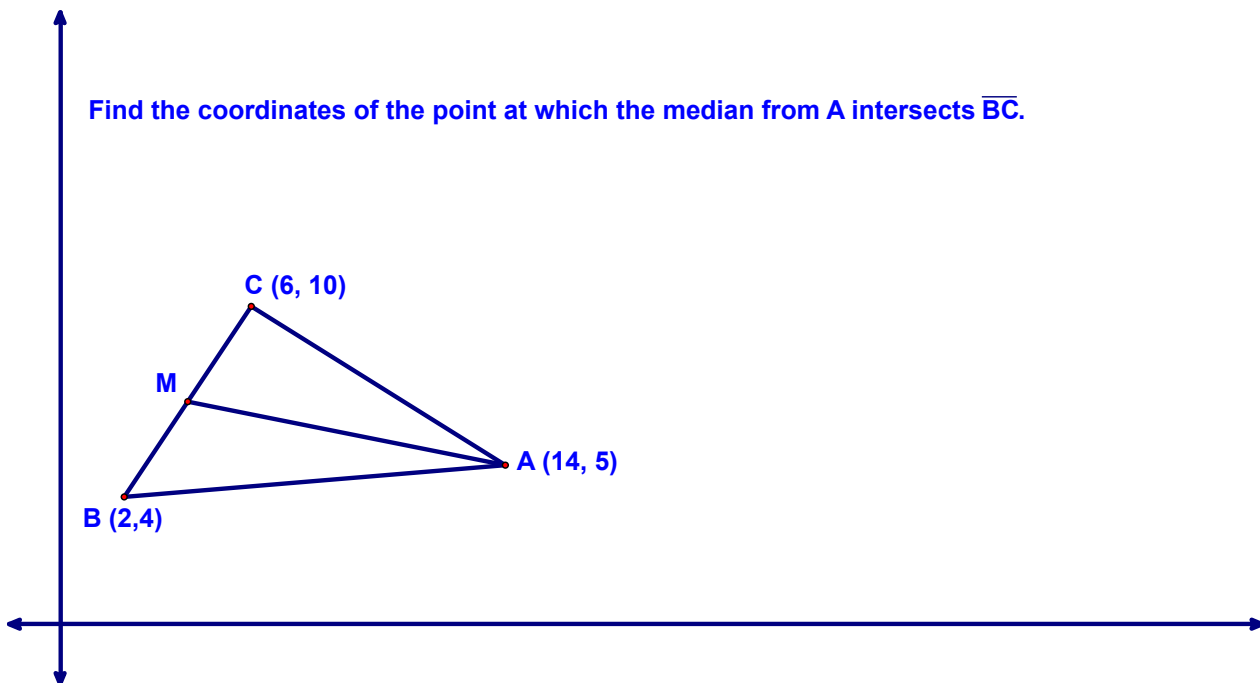
Here's one that's a little different. In this one you're told that one endpoint of a segment is (12,-8) and that the midpoint is (3,18) and your task is to find the other endpoint. To start this problem, let's see what this would look like on a graph:
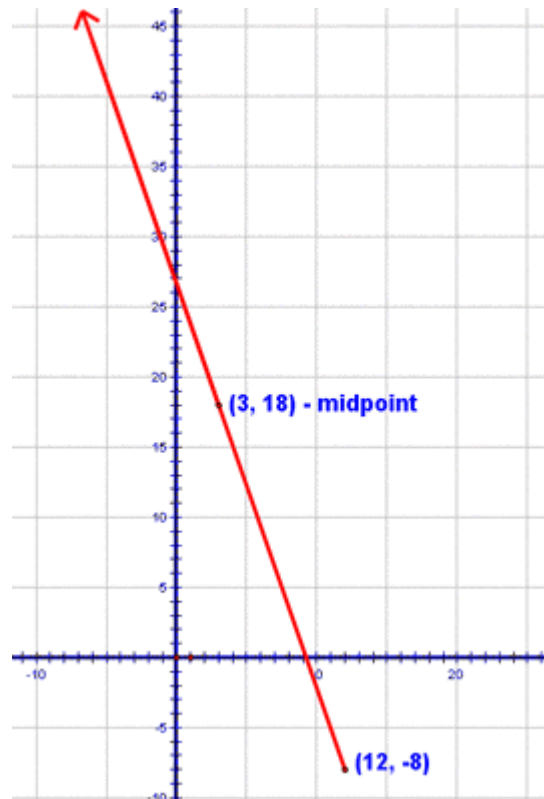

So, our other endpoint is somewhere along that red ray, right? There are two ways to approach this problem. The first is to think about the horizontal and vertical movements separately. Let's think about the horizontal movement first. To get from an x coordinate of 12 to one of 3, I have to go 9 spaces to the left. Now, wouldn't it make sense that to get from the midpoint to the other endpoint, I'd have to go 9 more spaces to the left? Let's see, 12 - 3 = 3 and 3 - 9 = -6. So, the x-coordinate of the other endpoint, must be -6!
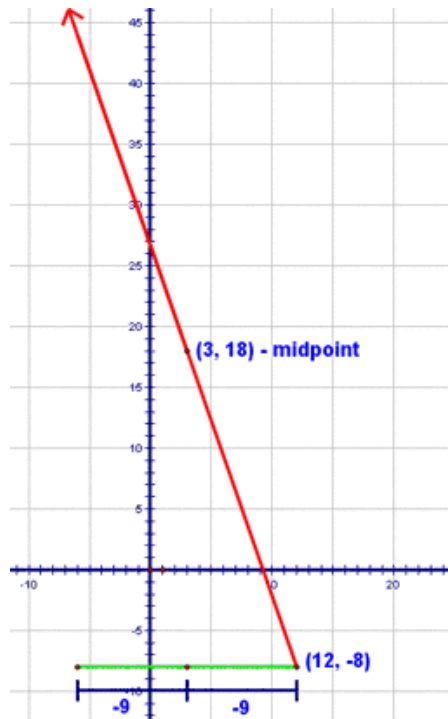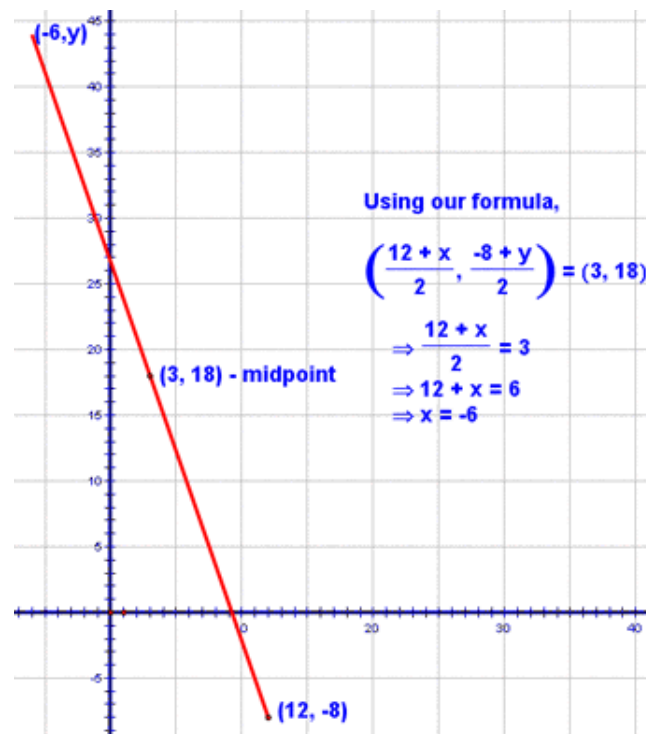
Let's apply the same process to finding the y-coordinate of the other endpoint. We're starting at a y coordinate of -8 and when we get to the midpoint, we're at a y-coordinate of 18. So...to get from -8 to 18, we add 26, right? That must mean that to get from the midpoint to the other endpoint, I've got to add 26 to the y-coordinate of the midpoint. So, 18 + 26 is 44!
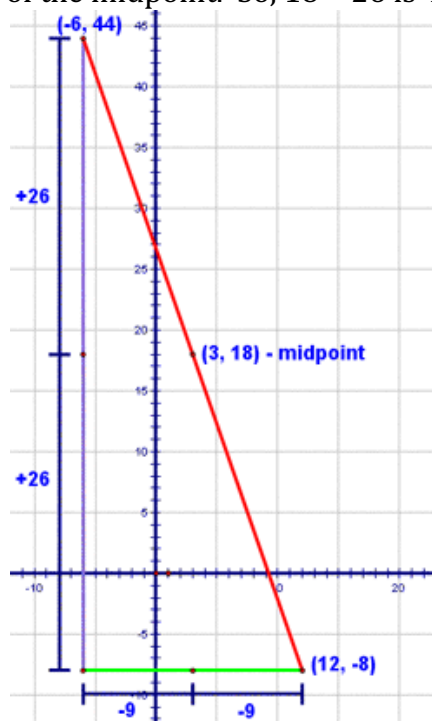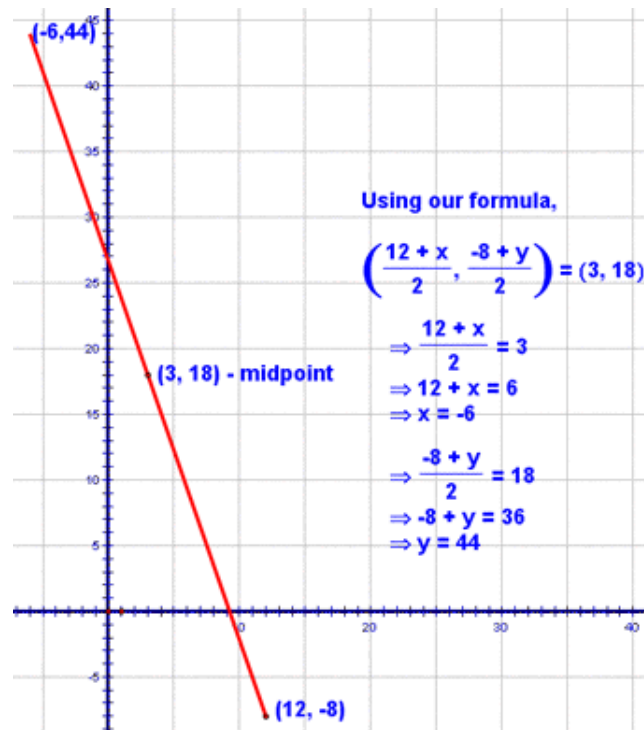
Alternatively, we could also solve this problem algebraically. Start by giving the other endpoint the coordinates (x, y). Then, use the midpoint formula:

(x,y)

Using our formula,

$$\left(\frac{12 + x}{2}, \frac{-8 + y}{2}\right) = (3, 18)$$

(3, 18) - midpoint

(12, -8)
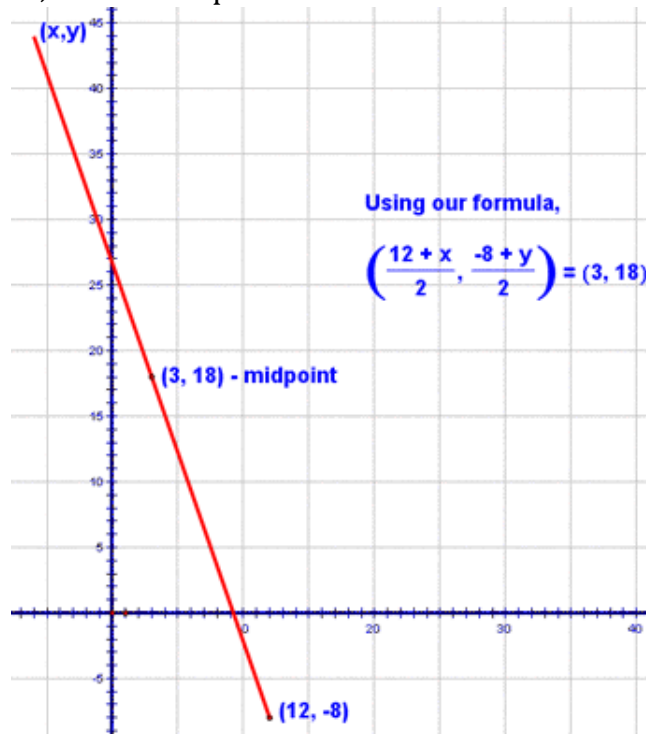
You should be able to see how the following is then true for the x-coordinate of the other endpoint:

(-6,y)

Using our formula,

$$\left(\frac{12 + x}{2}, \frac{-8 + y}{2}\right) = (3, 18)$$

$$\Rightarrow \frac{12 + x}{2} = 3$$

$$\Rightarrow 12 + x = 6$$

$$\Rightarrow x = -6$$

(3, 18) - midpoint

(12, -8)

The y-coordinate can be found in the same manner:

(-6,44)

Using our formula,

$$\left(\frac{12+x}{2}, \frac{-8+y}{2}\right) = (3, 18)$$

(3, 18) - midpoint

$$\Rightarrow \frac{12+x}{2} = 3$$
$$\Rightarrow 12+x = 6$$
$$\Rightarrow x = -6$$

$$\Rightarrow \frac{-8+y}{2} = 18$$
$$\Rightarrow -8+y = 36$$
$$\Rightarrow y = 44$$

(12, -8)

That's it for today...the midpoint stuff should be straight forward, but the detour proofs require a lot of practice, so make sure you do it...