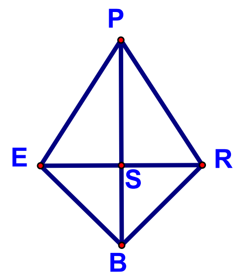
The Equidistance Theorems - Lesson 4.4

We will start today with the following warmup problem...It is a good application of the theorem we learned yesterday, so make sure you can do this!

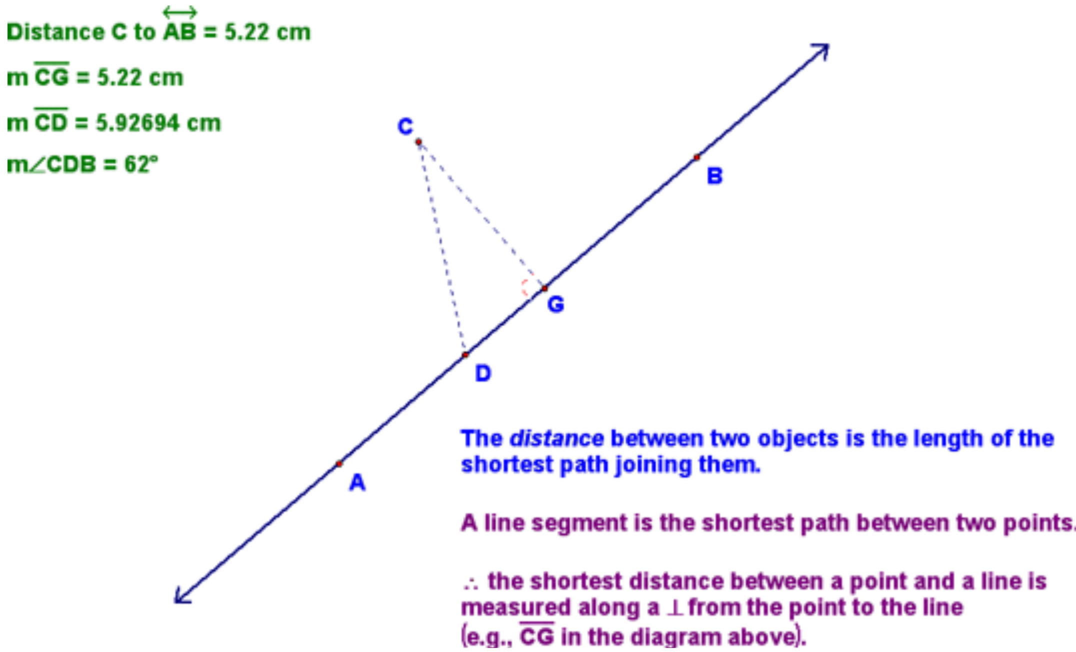
Given: $\overline{PE} \cong \overline{PR}$
$\overline{EB} \cong \overline{RB}$

Prove: $\overleftrightarrow{PB}$ is the $\perp$ bisector of $\overline{ER}$

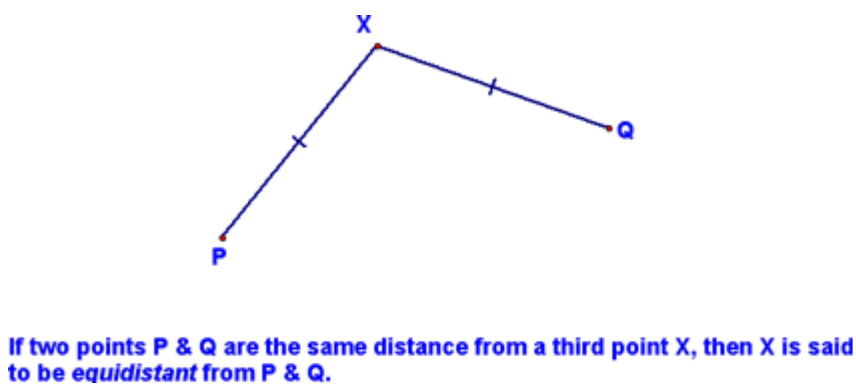


Statements	Reasons

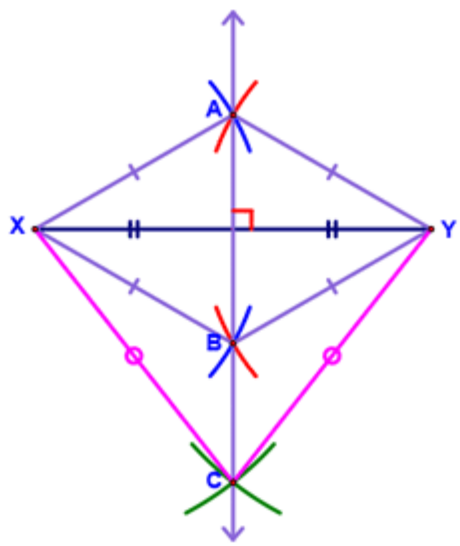
Now, let's talk about the the way to measure the distance between a point and a line...



and the term ***equidistant***:

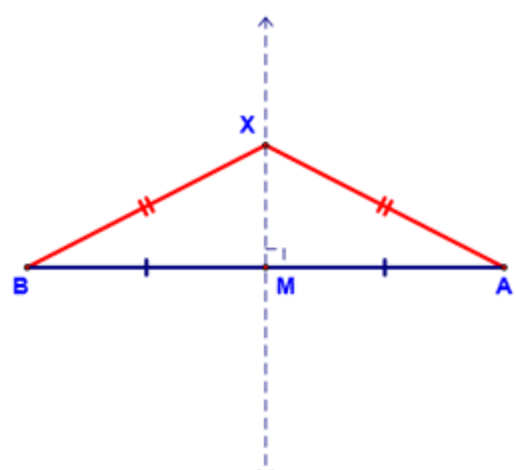


Now, let's consider two theorems that we will justify by doing a couple of constructions. For the first one, we'll start by defining the perpendicular bisector of a segment. You should remember that we learned how to construct this as show below (points A & B). You should recognize that we could have constructed point C, which is also equidistant from points X & Y, although at a different distance. This allows us to come to the conclusion shown in Theorem 24 (The Equidistance Theorem).



Theorem 25 **- If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of that segment (The Equidistance Theorem or ET).**

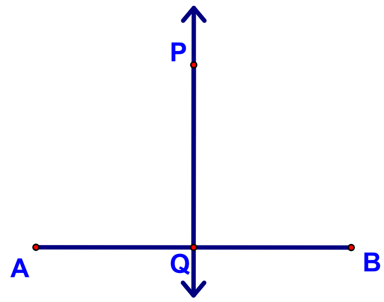
The converse of this theorem is also true...what if you already have the perpendicular bisector? What do you know about any point on it?



Theorem 26 **- If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment (The Converse of the Equidistance Theorem).**

As always, I think it is worth doing the formal proof these theorems. We proved ET in the warmup! Here, you should be able to come up with this diagram, givens, prove statement, etc. just from the theorem, but I'll give them to you so you can try the rest of the proof.

Theorem 26 **- If a point is on the ⊥ bisector of a segment, then it is equidistant from the endpoints of that segment (Converse of Equidistance Theorem)**



Given: $\overleftrightarrow{PQ}$ is the ⊥ bisector of $\overline{AB}$

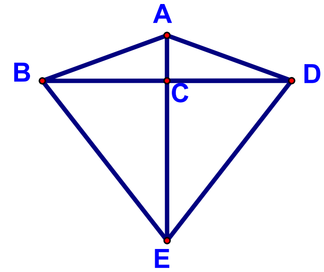
Prove: P is equidistant to A & B

Statements	Reasons

We'll concluded by using these theorems in a proof....gee...isn't it much shorter to do it this way, then to have to do all the steps that would be required otherwise (this proof is only 4 steps!!)? Also note that you'll often see these two theorems (ET and Converse of ET) used in combination like this in proofs.

Given: $\overline{AB} \cong \overline{AD}$
$\overline{BC} \cong \overline{CD}$

Prove: $\overline{BE} \cong \overline{ED}$



Statements	Reasons