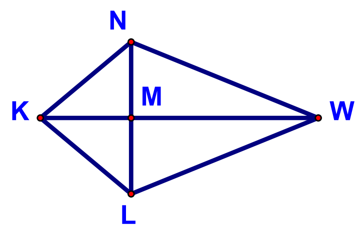
Proving that Lines are Parallel - Lesson 5-2

Today's warmup is a review of formal indirect proof…my hint is to not forget to phone home!!

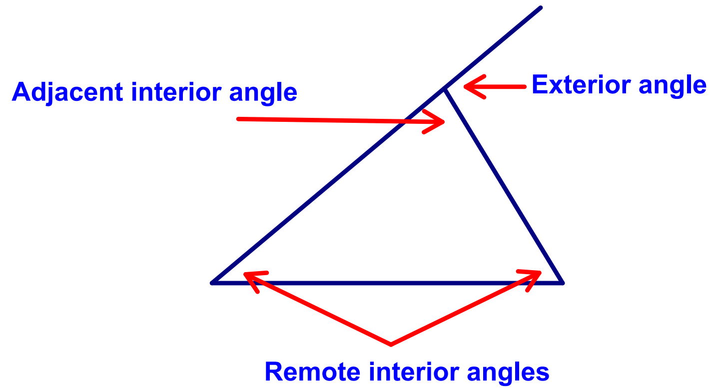
Given: M is the midpoint of $\overline{NL}$
$\overline{KN}$ is not ≅ to $\overline{KL}$

Prove: $\overline{NW}$ is not ≅ to $\overline{LW}$



Statements	Reasons

Today, we're going to cover a number of theorems regarding how to prove that lines are parallel. However, in order to prove these, we have to start by understanding what an exterior angle is:



You should be able to recognize that there are 6 exterior angles for every triangle. Now, let's talk about a number of theorems that can be proved based on the first (#29). We're not going to prove Theorem #29, but you should be able to find a proof online if you're curious.

Theorem 29 **- The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle (Exterior Angle Inequality Theorem).**

Theorem 30 **- If two lines are cut by a transversal such that two alternate interior angles are congruent, the lines are parallel (Alt. int. $\angle$s $\cong$ $\Rightarrow$ $\parallel$ lines or AIP).**

Theorem 31 **- If two lines are cut by a transversal such that two alternate exterior angles are congruent, the lines are parallel (Alt. ext. $\angle$s $\cong$ $\Rightarrow$ $\parallel$ lines or AEP).**

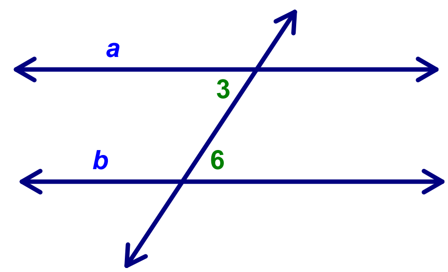
Theorem 32 **- If two lines are cut by a transversal such that two corresponding angles are congruent, the lines are parallel (Corr. $\angle$s $\cong$ $\Rightarrow$ $\parallel$ lines or CAP).**

Assuming that the Exterior Angle Inequality Theorem (#29) is true, let's prove AIP. The others are very similar.

Theorem 30 **- If two lines are cut by a transversal such that two alternate interior angles are congruent, the lines are parallel (Alt. int. ∠s ≅ ⇒ || lines or AIP).**

Given: ∠3 ≅ ∠6

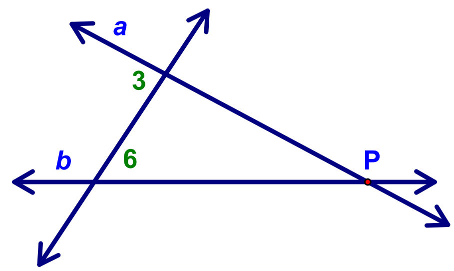
Prove: a || b



Assume that a is not || to b.

⇒ that *a* and *b* must intersect at some point P.
∠3 is an exterior angle of the Δ formed, so by the Exterior Angle Inequality Theorem, m∠3 > m∠6.

But this contradicts the given fact that ∠3 ≅ ∠6.
∴ The assumption was false and the lines are ||



You should be able to see how you could prove AEP & CAP by a similar method.

Now, let's understand a number of other theorems that are related:

Theorem 33 **- If two lines are cut by a transversal such that two interior angles on the same side of the transversal are supplementary, the lines are parallel (SSISP).**

Theorem 34 **- If two lines are cut by a transversal such that two exterior angles on the same side of the transversal are supplementary, the lines are parallel (SSESP).**

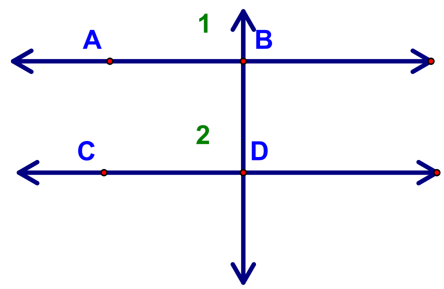
Theorem 35 **- If two coplanar lines are perpendicular to a third line, they are parallel.**

Let's wrap up by doing an example (which is the proof for Theorem 35). You should be able to prove this using one of AIP, AEP, or CAP!!

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{BD}$

$\overleftrightarrow{CD} \perp \overleftrightarrow{BD}$

Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



Statements	Reasons