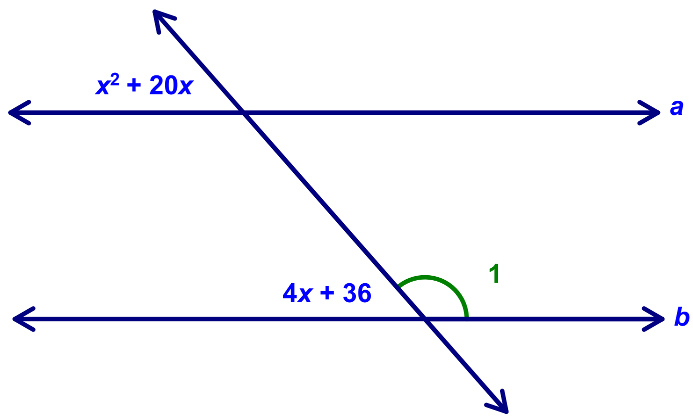
Congruent Angles Associated with Parallel Lines - Lesson 5-3

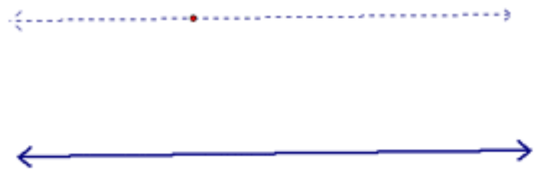
Here's our warmup!

$a \parallel b$

Find m∠1

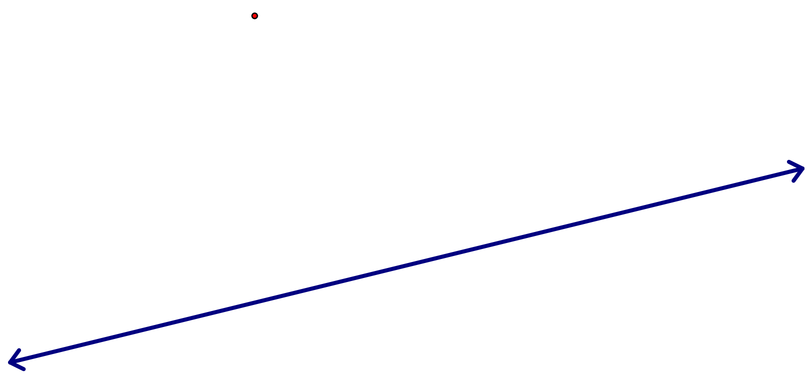


Today, we're starting by discussing the Parallel Postulate:

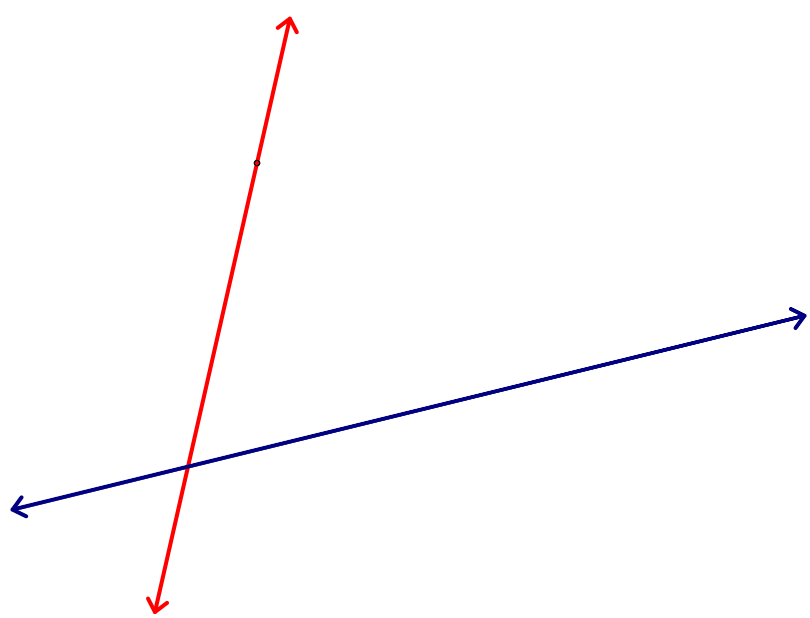


Parallel Postulate: Through a point not on a line there is exactly one parallel to the given line.

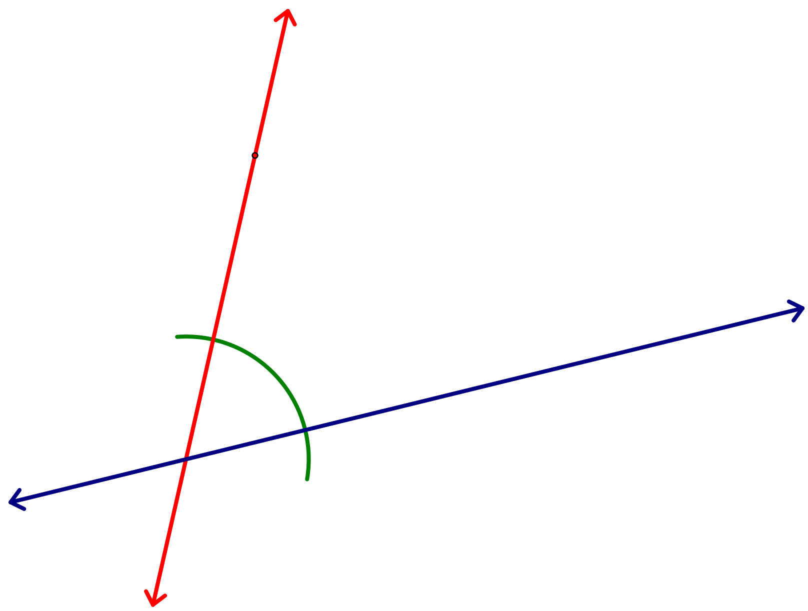
We can use this postulate along with the AIP theorem to construct parallel lines using the Alternate Interior Angles Method. For this, you start with a given line and a given point not on that line:



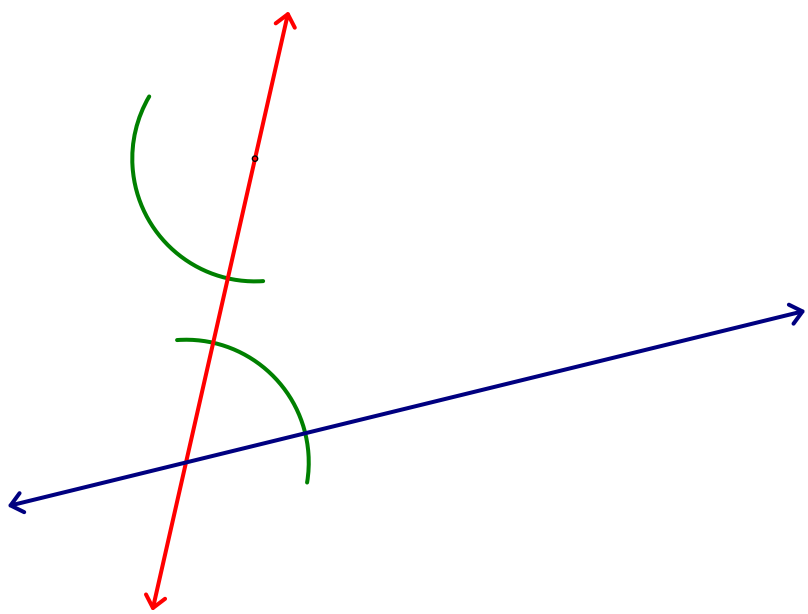
Next, you use your straight edge to construct a second line that goes through both the given point and the given line. This can be done at any angle.



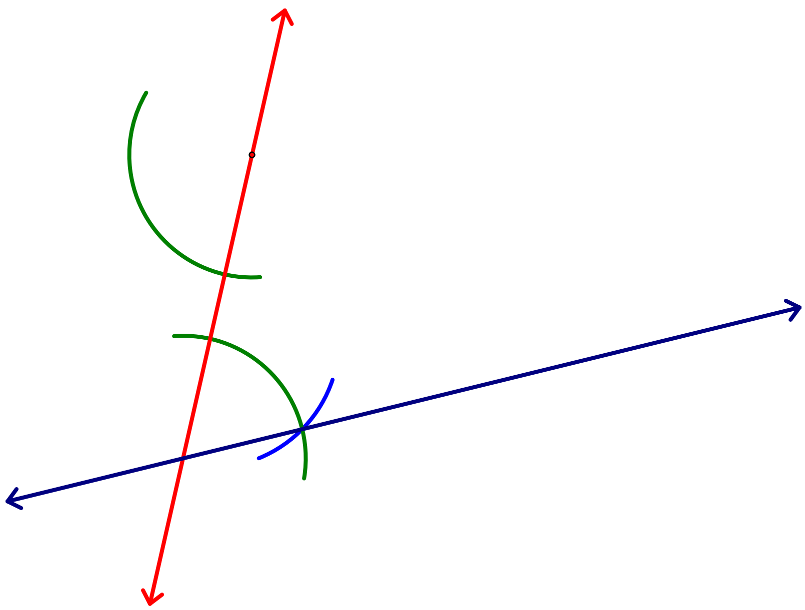
Now, center your compass at the point where these two lines intersect and draw an arc that crosses both legs of the angle.



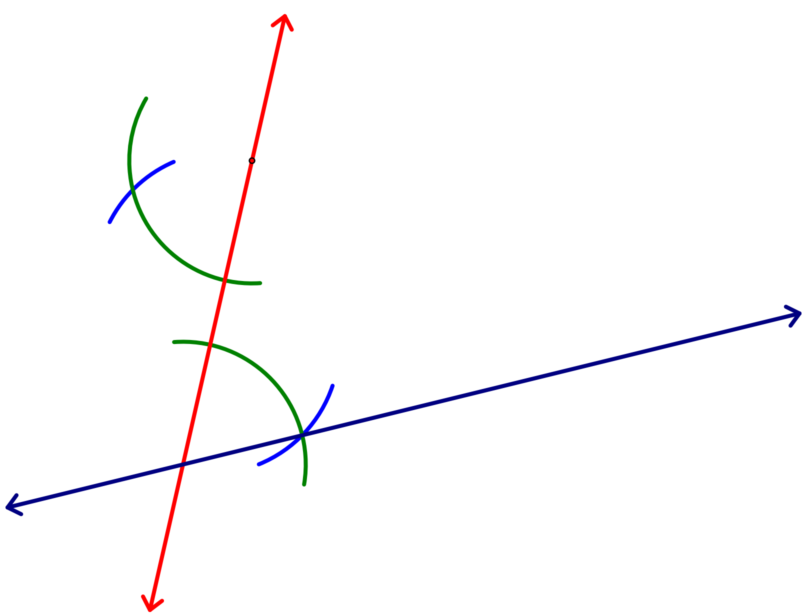
Using the same radius as in the previous step, center your compass at the given point and construct an arc that crosses the red line and the place where you think the parallel line will be.



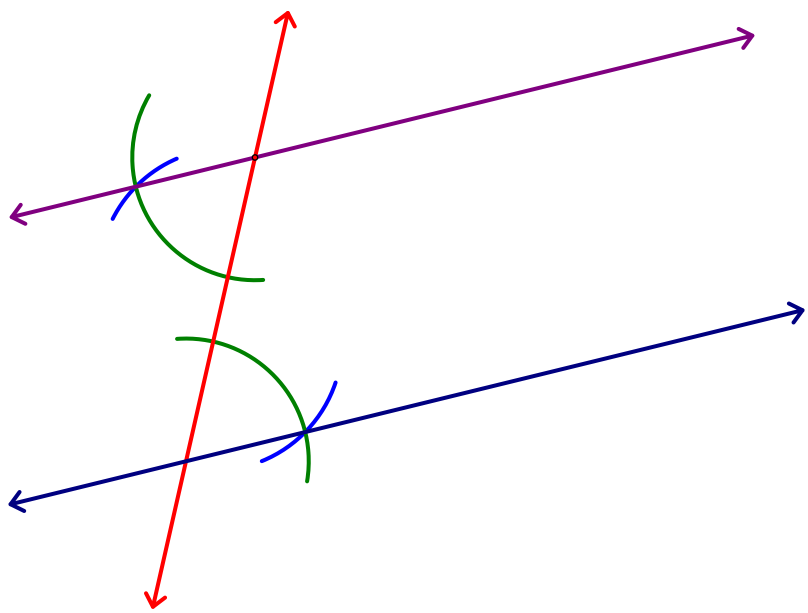
Now, center your compass at the point where the first arc crosses the red line. Set the radius to the distance from here to the point where the first arc crosses the given line. Draw an arc (the only reason to do this is to make sure you have the distance correctly set).



Using the same radius as the previous step, center your compass at the point where the second green arc crosses the red line. Make an arc that crosses the second green arc.



You're there! Now you just have to use your straight edge to connect this new point with the given point. You now have parallel lines (the given line and the purple line). Do you see how we created them? Right...we constructed congruent alternate interior angles!

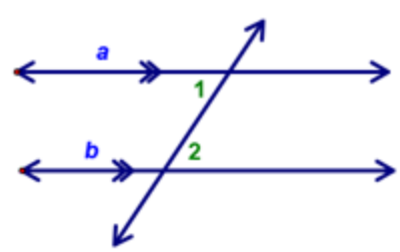


The Parallel Postulate also helps us to prove the following theorem. The key thing here to understand is that if angle 1 is not congruent to angle 2, then there has to be a line other than a that goes through point P that is parallel to line b (because of the Parallel Postulate). Once you understand that, the rest follows.

Theorem 36: **If two parallel lines are cut by a transversal, each pair of alternate interior angles are congruent (||lines ⇒ ≅ alt. int. ∠s or PAI).**

Given: a||b

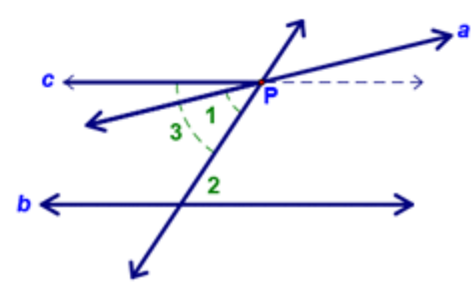
Prove: ∠1 ≅ ∠2



Assume that ∠1 is not ≅ to ∠2

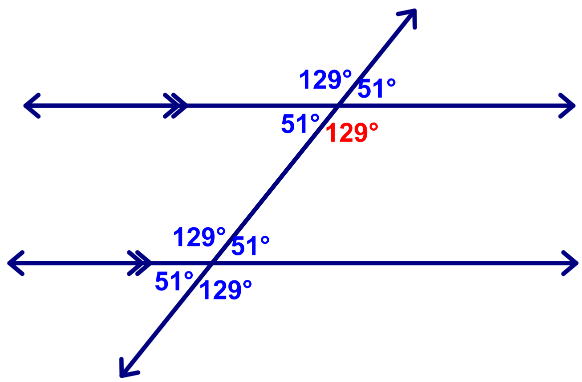
⇒ there must be another line, c, that intersects the transversal at P to form an ∠3 ≅ ∠2.
Now alt. int. ∠s ≅ ⇒ || lines (AIP), so *c*||*b*

But this means that *b* is parallel to two lines in the plane at point P which violates the parallel postulate.
∴ Our assumption was false and ∠1 ≅ ∠2



You should now recognize that parallel lines with a transversal really give us a lot of information. We started with the following:

Theorem 37: **If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.**



And then moved on to these theorems (most of which are the converses of ones we did yesterday).

Theorem 38: **If two parallel lines are cut by a transversal, each pair of alternate exterior angles are congruent (PAE).**

Theorem 39: **If two parallel lines are cut by a transversal, each pair of corresponding angles are congruent (PCA).**

Theorem 40: **If two parallel lines are cut by a transversal, each pair of interior angles on the same side of the transversal are supplementary (PSSIS).**

Theorem 41: **If two parallel lines are cut by a transversal, each pair of exterior angles on the same side of the transversal are supplementary (PSSES).**

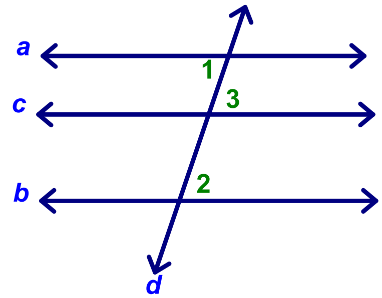
Theorem 42: **In a plane, if a line is $\perp$ to one of two $\parallel$ lines, it is $\perp$ to the other.**

Theorem 43: **If two lines are $\parallel$ to a third line, they are $\parallel$ to each other (Transitive Property of $\parallel$ lines).**

We'll conclude by proving Theorem 43. This should be pretty straight forward for you by now!

Given: a||b
b||c

Prove: a||c



Statements	Reasons