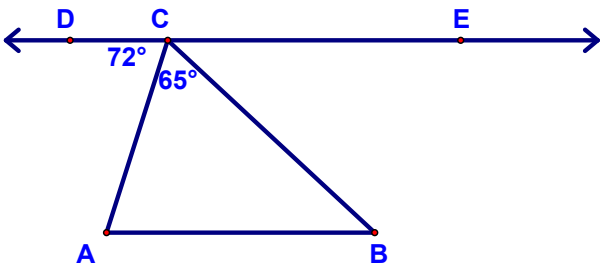
Triangle Application Theorems - Lesson 7-1

Here's the warmup!

Given: $\overleftrightarrow{DE} \parallel \overline{AB}$
$m\angle ACB = 65°$
$m\angle ACD = 72°$

Find: $m\angle A$ and $m\angle B$

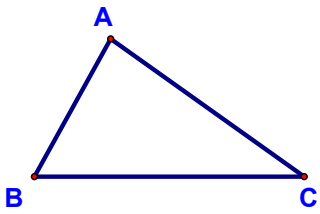


Today, we will finally prove that the sum of the angles of a triangle is 180 degrees!! This is clearly something you've know for quite a while (a couple of years at least!), but was also something we couldn't prove until recently. With what we know about parallel lines and alternate interior angles, it's pretty straight forward:

Theorem 49: **The sum of the measures of the three angles of a triangle is 180°**

Given: $\triangle ABC$

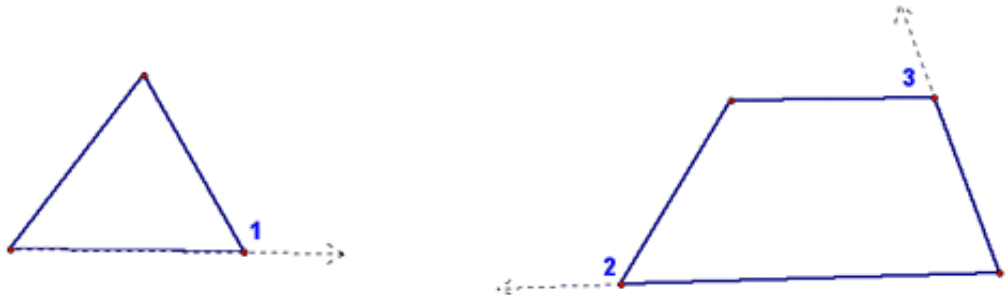
Prove: $m\angle A + m\angle B + m\angle C = 180°$



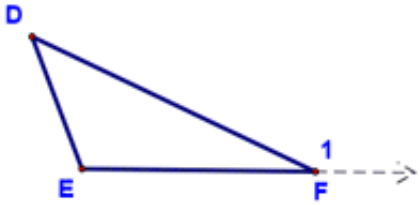
Statements	Reasons

Next, we're going to formally define exterior angles and show that the measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles:

An *exterior angle* of a polygon is an angle that is adjacent to and supplementary to an interior angle of the polygon.



Theorem 50: **The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.**



So, for the diagram above, m∠1 = m ∠D + m∠ E.

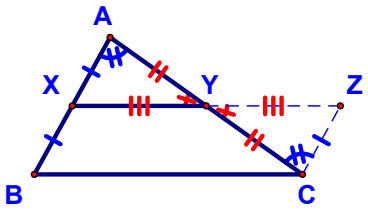
We wrapped up with the following theorem and its proof...very cool stuff here!!

Theorem 51: **A segment joining the midpoints of two sides of a Δ is || to the third side and its length is one-half the length of the third side (Midline Theorem).**

Given: X & Y are midpoints

Prove: $\overline{XY} \parallel \overline{BC}$

$XY = \frac{1}{2}(BC)$



	Statements	Reasons
S	1. Extend $\overline{XY}$ through Y to a point Z so that $\overline{XY} \cong \overline{YZ}$	1. Auxiliary Lines
	2. Draw $\overline{ZC}$	2. Auxiliary Lines
	3. X & Y are midpoints	3. Given
S	4. $\overline{AY} \cong \overline{CY}$	4. Defn of midpoint
A	5. $\angle AYX \cong \angle ZYC$	5. VAT
	6. $\Delta AYX \cong \Delta CYZ$	6. SAS (1, 5, 4)
	7. $\angle XAY \cong \angle ZCY$	7. CPCTC
	8. $\overline{AB} \parallel \overline{ZC}$	8. AIP
	9. $\overline{AX} \cong \overline{ZC}$	9. CPCTC
	10. $\overline{AX} \cong \overline{XB}$	10. Defn of midpoint
	11. $\overline{XB} \cong \overline{ZC}$	11. Transitive Property
	12. XBCZ is ▱	12. If a Quad has 1 pair of opp. sides both ∥and ≅, it is a ▱
	13. $\overline{XY} \parallel \overline{BC}$	13. Defn. of ▱
	14. $\overline{XZ} \cong \overline{BC}$	14. Opp. sides of a Y are ≅
	15. XY = YZ	15. If segments are ≅, then they have the same measure
	16. XZ = XY + YZ	16. Assumed from diagram
	17. XZ = 2(XY)	17. Substitution Property of Equality (15→16)
	18. $XY = \frac{XZ}{2}$	18. Division Property of Equality
	19. XZ = BC	19. If segments are ≅, then they have the same measure
	20. $XY = \frac{BC}{2}$	20. Substitution Property of Equality (19→18)