Ratio and Proportion - Lesson 8-1

Here's today's warmup!

Two supplementary angles are in the ratio of 3 to 5. Find the measure of the larger angle.

Today, we'll begin to look at similarity. However, before we start, we have to review ratio, proportion, and a few relevant theorems:

A *ratio* is an expression that compares two number by division (a *quotient* of two numbers).

e.g., the ratio of boys to girls in the class vs. the ratio of girls to boys.

Ratios can be simplified:

(simplify $\frac{\text{2 ft.}}{\text{12 in.}}$ and $\frac{\text{5 nickels}}{\text{2 quarters}}$).

A *proportion* is a statement of equality between two or more ratios.

e.g., $\frac{3}{4} = \frac{6}{8}$

Proportions can be solved. Please solve:

$\frac{3}{5} = \frac{8}{x}$, $\frac{3-x}{30} = \frac{6}{18}$, and $\frac{30}{m+1} = \frac{10}{3m-5}$

The following theorems should be familiar to you (although probably not in theorem form!)...the first one is simply cross-multiplication and the second basically means that you can always exchange the means in a proportion and/or "flip" both sides of a proportion.

Theorem 58: **In a proportion, the product of the means is equal to the product of the extremes (Means-Extremes Products Theorem).**

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc$$

Theorem 59: **If the product of a pair of nonzero numbers is equal to the product of another pair of nonzero number, then either pair of numbers may be made the extremes, and the other pair the means, of a proportion (Means-Extremes Ratio Theorem).**

$$\text{If } pq = rs, \text{ then } \frac{p}{r} = \frac{s}{q}, \frac{p}{s} = \frac{r}{q}, \text{ and } \frac{r}{p} = \frac{q}{s}$$

You should be very comfortable messing around with all these ratios and proportions. Remember to always leave your answers as fully reduced improper fractions...I really don't like (and will take points off for) mixed numbers and decimal approximations of improper fractions!!

We wrapped up the day by defining the *arithmetic mean* and the *geometric mean* of two numbers:

If the means in a proportion are equal, either mean is called a *geometric mean*, or *mean proportional*, between the extremes.

The average of two numbers is called the *arithmetic mean*.

e.g., Find the arithmetic and geometric mean (a.k.a mean proportional) between 32 and 2.

Arithmetic Mean (Average)

$$\frac{32 + 2}{2} = 17$$

Geometric Mean (Mean Proportional)

$$\frac{2}{x} = \frac{x}{32}$$

$$x^2 = 64$$

$$x = \pm 8$$