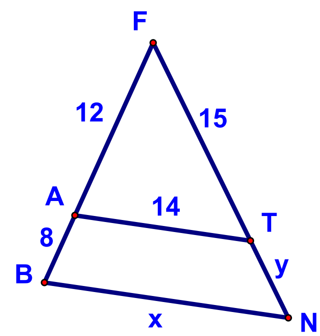
Three Theorems Involving Proportions - Lesson 8-5

Here's the warmup!

Given: $\overline{AT} \parallel \overline{BN}$
FA = 12, FT = 15,
AT = 14, AB = 8

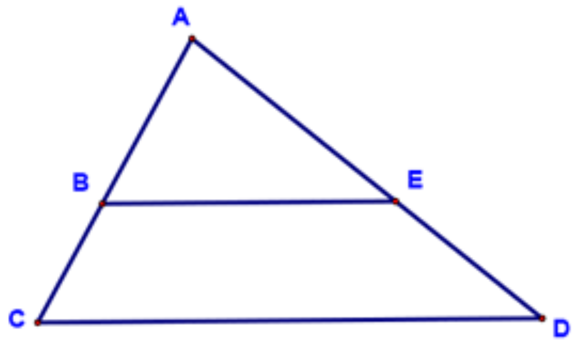
Find: BN and TN



Today, we're going to learn some theorems that prove useful when solving problems involving similar triangles. We'll start with the side-splitter theorem:

Theorem 64

If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally (Side-Splitter Theorem).



Given: $\overline{BE} \parallel \overline{CD}$

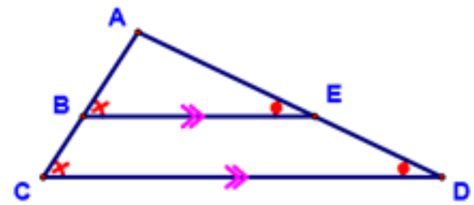
Conclusion: $\frac{AB}{BC} = \frac{AE}{ED}$

For those of you who are interested, here's the proof for the side-splitter theorem:

Theorem 64: **If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally (Side-Splitter Theorem).**

Given: $\overline{BE} \parallel \overline{CD}$

Prove: $\frac{AB}{BC} = \frac{AE}{ED}$



Statements	Reasons
1. $\overline{BE} \parallel \overline{CD}$	1. Given
AA 2. $\angle ABE \cong \angle C$; $\angle AEB \cong \angle D$	2. PCA
3. $\triangle ABE \sim \triangle ACD$	3. AA ~ (2, 2)
4. $\frac{AB}{AC} = \frac{AE}{AD}$	4. CSSTP
5. AC = AB + BC; AD = AE + ED	5. Assumed from Diagram
6. $\frac{AB}{AB + BC} = \frac{AE}{AE + ED}$	6. Substitution (5 → 4)
7. AB(AE + ED) = AE(AB + BC)	7. MEPT
8. (AB·AE)+(AB·ED) = (AE·AB)+(AE·BC)	8. Distribution Property
9. (AB · ED) = (AE · BC)	9. Subtraction Property of Equality
10. $\frac{AB}{BC} = \frac{AE}{ED}$	10. MERT

Next, we can apply the side-splitter theorem to parallel lines:

Theorem 65

If three or more parallel lines are intersected by two transversals, the parallel lines divide the transversals proportionally.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

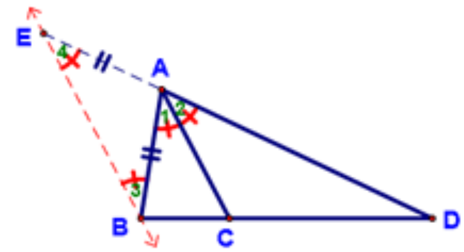
Conclusion: $\dfrac{AC}{CE} = \dfrac{BD}{DF}$

Finally, we'll look at the proportions we can set up if we have an angle bisector...this is a nice proof, huh?

Theorem 66: **If a ray bisects an ∠ of a △, it divides the opposite side into segments that are proportional to the adjacent sides (Angle Bisector Theorem).**

Given: △ABD
$\overrightarrow{AC}$ bis. ∠BAD

Prove: $\frac{BC}{CD} = \frac{AB}{AD}$



Statements	Reasons
1. △ABD	1. Given
2. $\overrightarrow{AC}$ bis. ∠BAD	2. Given
3. ∠1 ≅ ∠2	3. Defn. of ∠ bis.
4. Draw through B the line ∥ to $\overline{AC}$	4. Parallel Postulate
5. Extend $\overline{DA}$ to intersect $\overline{EB}$ at E	5. Auxiliary Lines
6. $\frac{BC}{CD} = \frac{EA}{AD}$	6. Side-Splitter Theorem
7. ∠1 ≅ ∠3	7. PAI
8. ∠2 ≅ ∠4	8. PCA
9. ∠3 ≅ ∠4	9. Transitive Property of ≅ ∠s (3, 7, 8)
10. $\overline{EA} \cong \overline{AB}$	10. Converse of ITT
11. EA = AB	11. Definition of ≅ Segments
12. $\frac{BC}{CD} = \frac{AB}{AD}$	12. Substitution (11 → 6)

We'll end by solving the following problem...the first proportion is straight forward, but don't get fooled (again...) by the second one!

Given: $\overline{BE} \parallel \overline{CD}$
Lengths as shown

Find: ED
CD