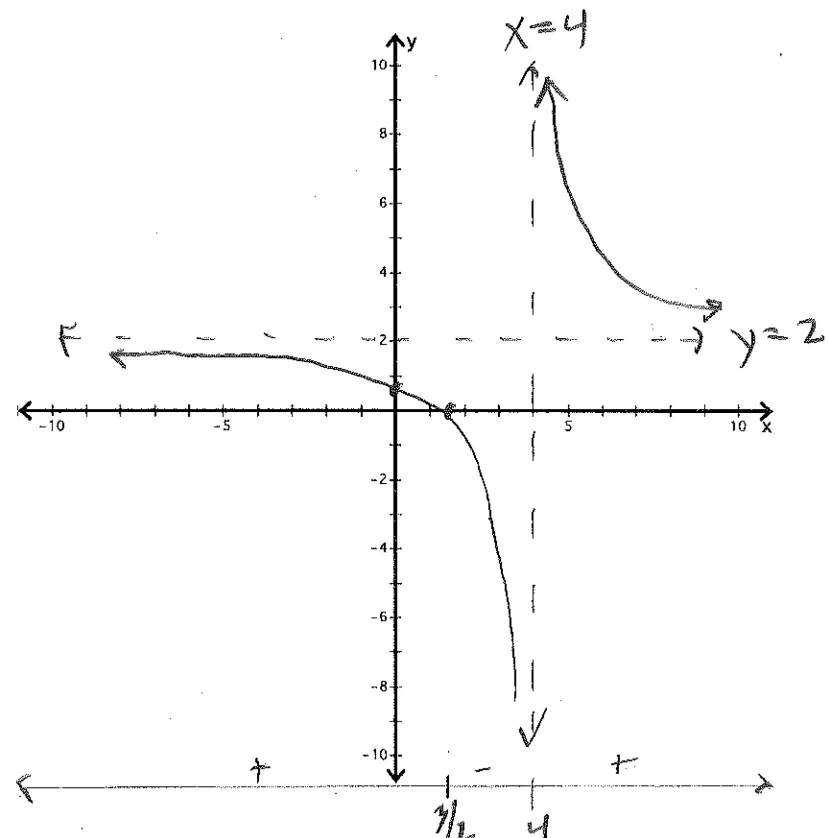
Sections 2.6 & 2.7 – I.C.E #2

Find all horizontal, slant, and vertical asymptotes for each function. Be sure to state your answers as equations of lines. Also find all x and y-intercepts and draw a sketch of the graph. Label where the asymptotes and the intercepts are located on your graph.

1) $f(x)=\dfrac{2x-3}{x-4}$



Hole? no

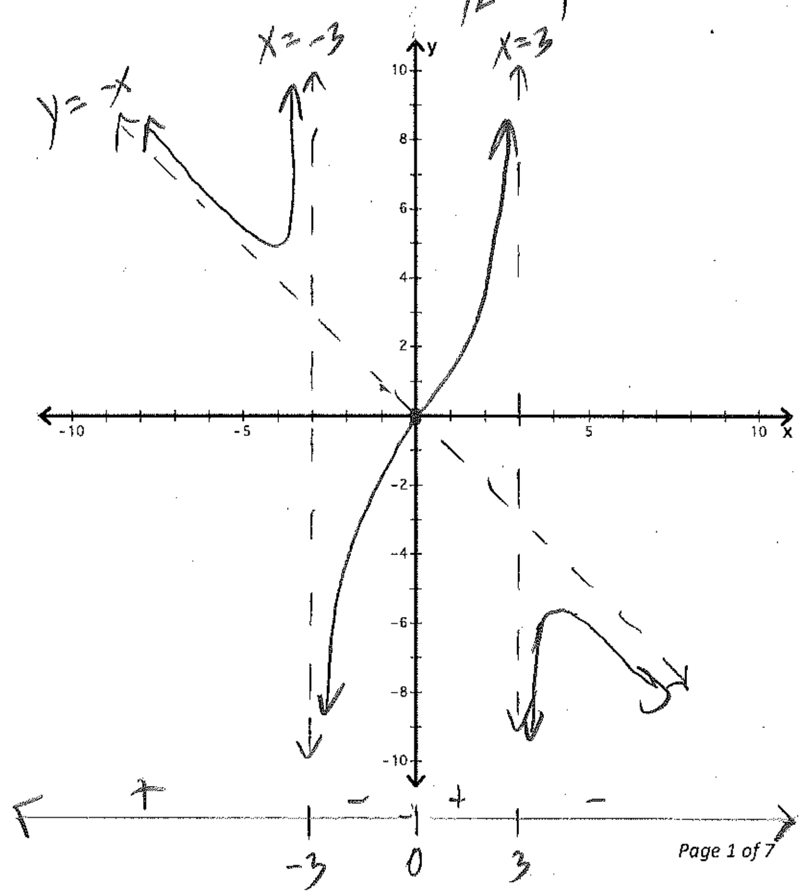
VA: $x=4$

HA or SA: $y=2$

x-int: $(3/2, 0)$

y-int: $(0, 3/4)$

2) $f(x)=\dfrac{-x^3}{x^2-9}=\dfrac{-x^3}{(x-3)(x+3)}$



Hole? no

VA: $x=3,\ x=-3$

HA or SA: $y=-x$

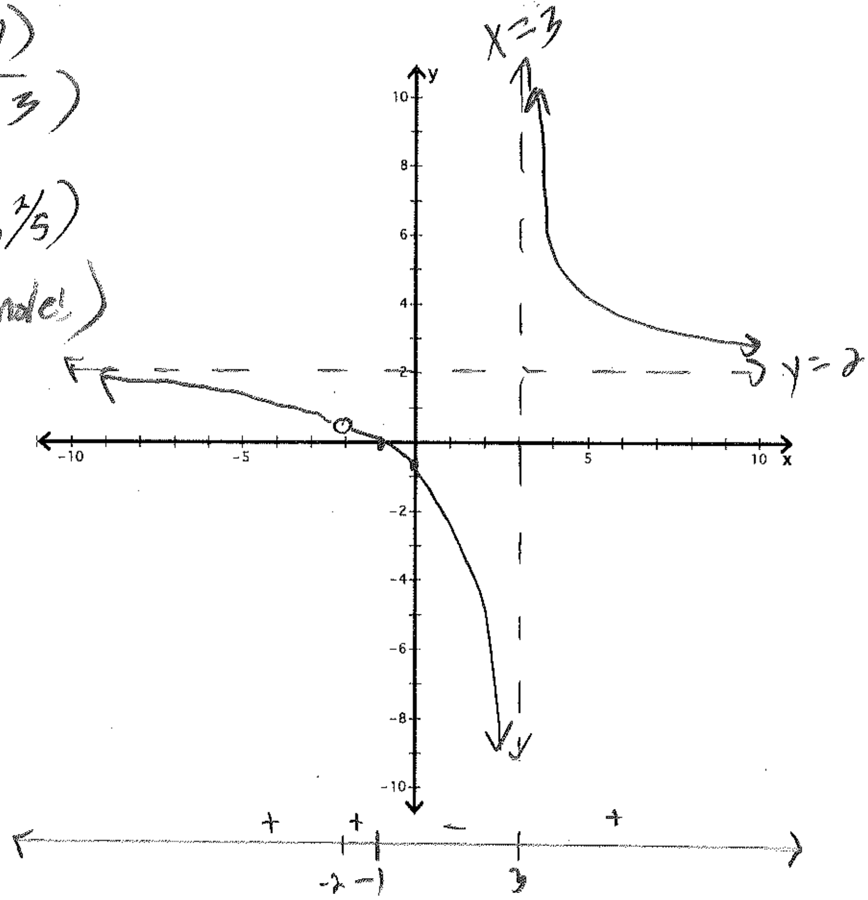
x-int: $(0,0)$

y-int: $(0,0)$

$$\begin{array}{r}
-x \\
x^2-9\,\overline{)\,-x^3+0x^2+0x+0} \\
-(-x^3+0x^2+9x) \\
\hline
-9x
\end{array}$$

Sections 2.6 & 2.7 – I.C.E #2

3) $f(x)=\dfrac{2x^2+6x+4}{x^2-x-6} = \dfrac{2(x+2)(x+1)}{(x+2)(x-3)}$



Hole? yes: $x=-2$ $(-2, \frac{2}{5})$

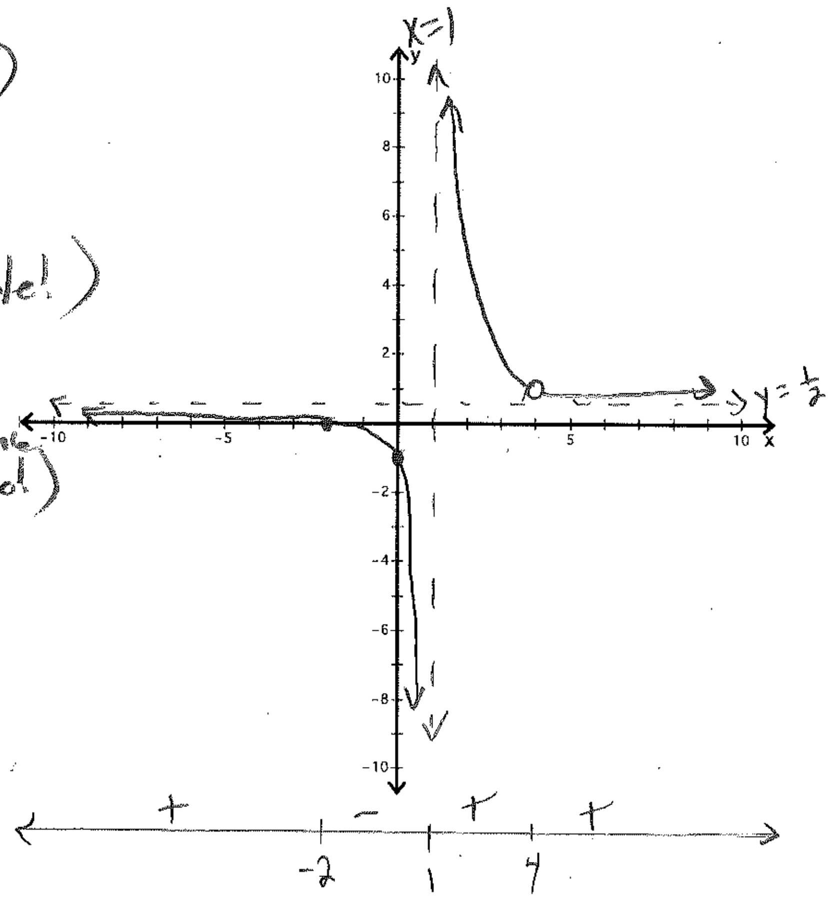
VA: $x=3$ (not -2 since hole!)

HA or SA: $y=2$

x-int: $(-1,0)$

y-int: $(0,-\frac{2}{3})$

4) $f(x)=\dfrac{x^2-2x-8}{2x^2-10x+8} = \dfrac{(x-4)(x+2)}{2(x-4)(x-1)}$



Hole? yes: $x=4$ $(4, \frac{3}{5})$

VA: $x=1$ (not 4 since hole!)

HA or SA: $y=\frac{1}{2}$

x-int: $(-2,0)$ [not $(4,0)$ since hole!)

y-int: $(0,-1)$

Sections 2.6 & 2.7 – I.C.E #2

5) $g(x)=\dfrac{5}{x^2-16} = \dfrac{5}{(x+4)(x-4)}$

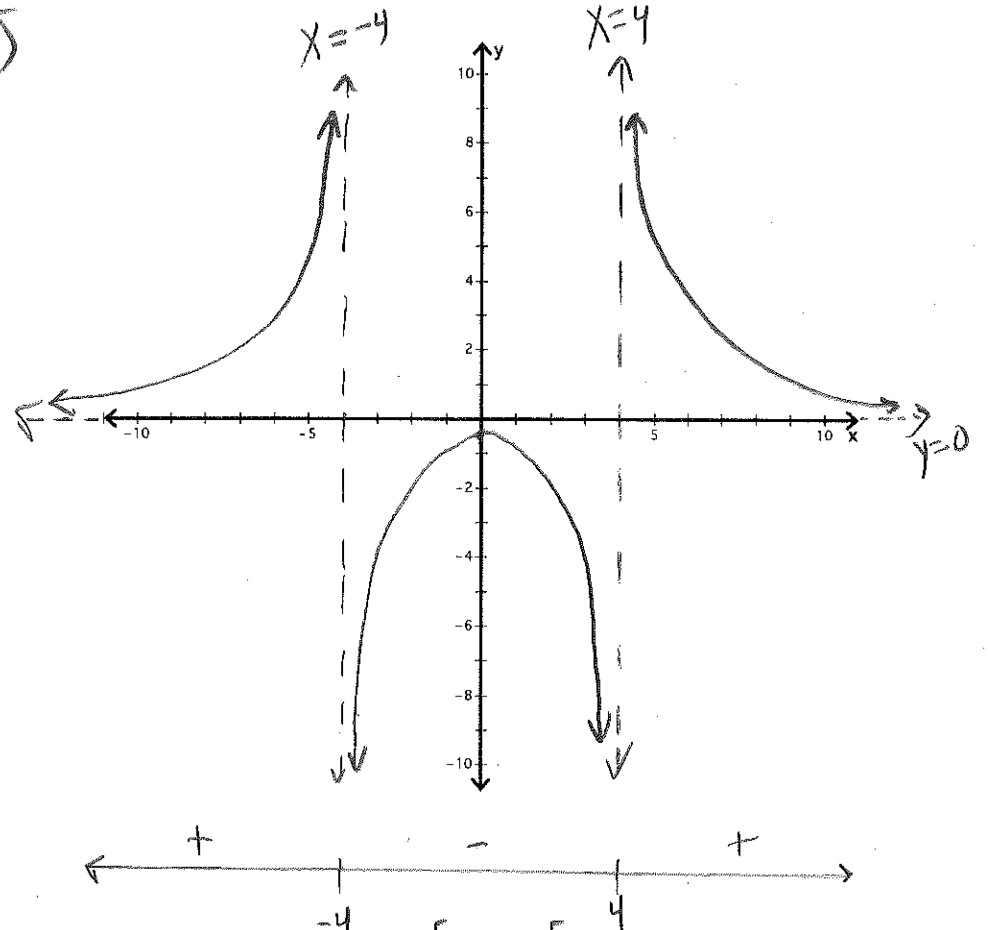
Hole? no

VA: $x=4, x=-4$

HA or SA: $y=0$

x-int: none

y-int: $(0, -5/16)$



6) $h(x)=\dfrac{-4x}{x^2-2} = \dfrac{-4x}{(x+\sqrt{2})(x-\sqrt{2})}$

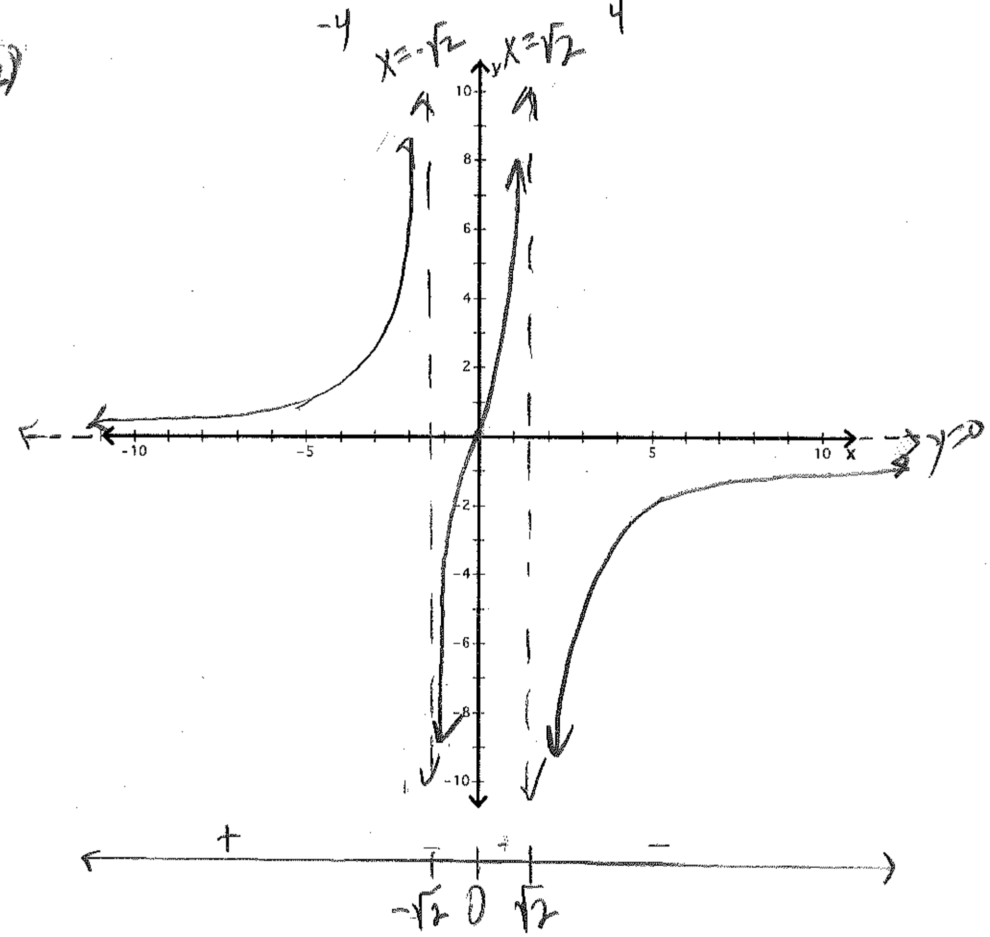
Hole? no

VA: $x=\sqrt{2}, x=-\sqrt{2}$

HA or SA: $y=0$

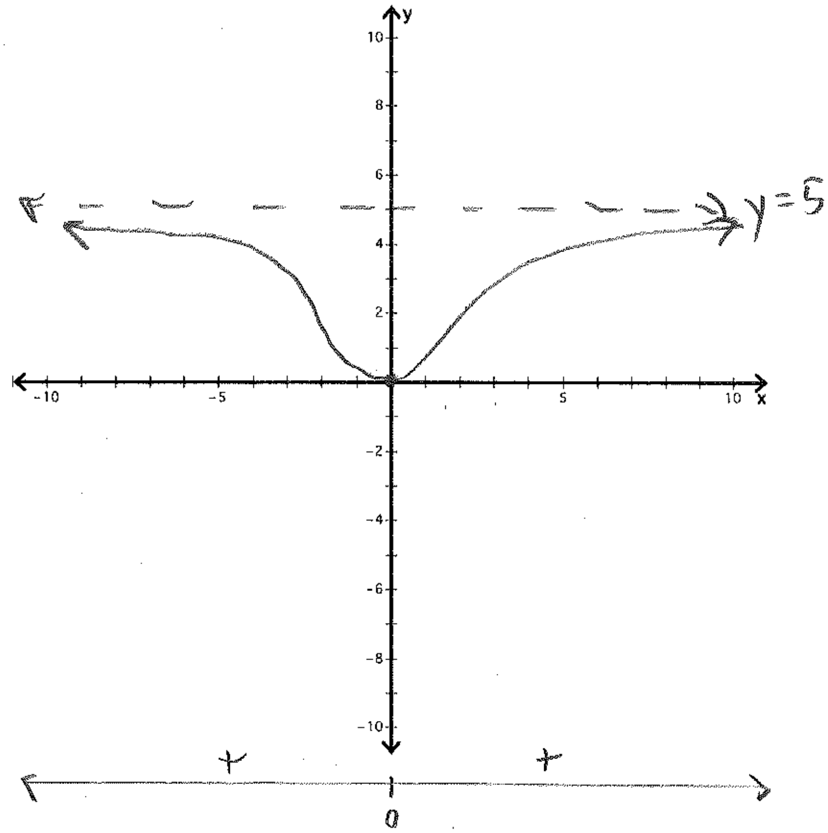
x-int: $(0,0)$

y-int: $(0,0)$



Sections 2.6 & 2.7 – I.C.E #2

7) $f(x)=\dfrac{3x^2}{x^2+5}$



Hole? no

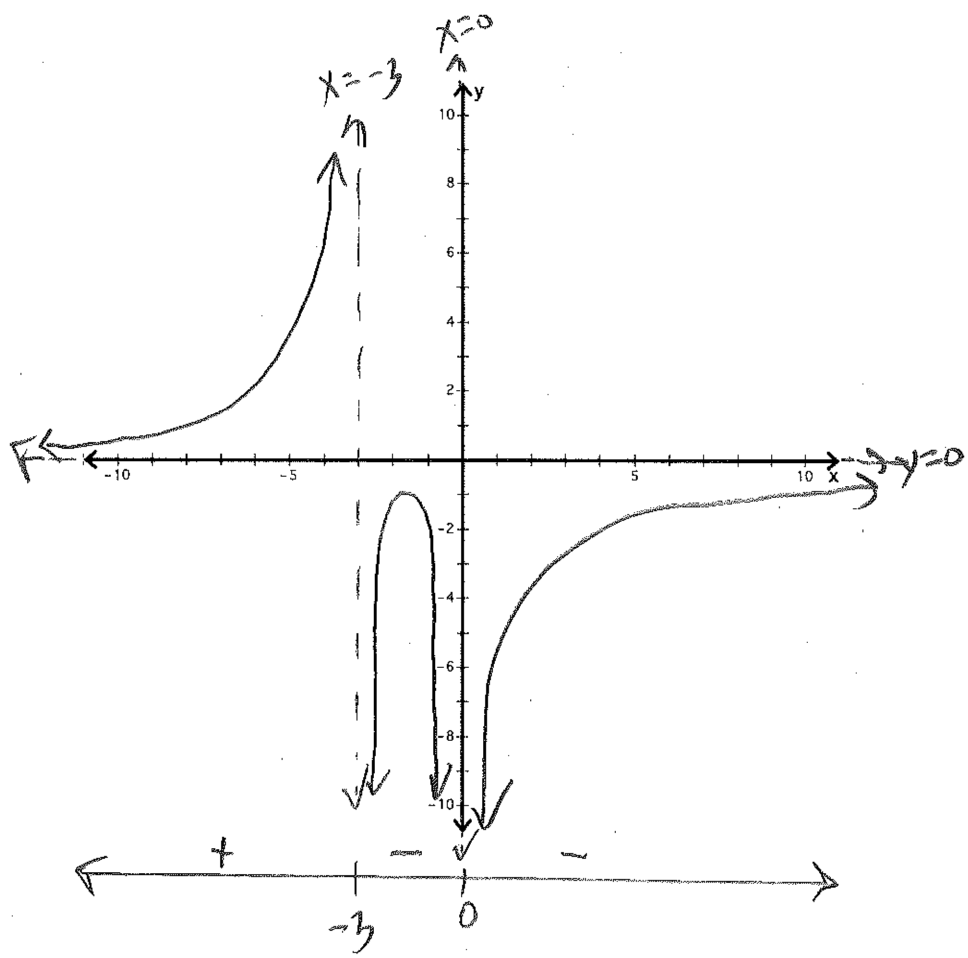
VA: none

HA or SA: y = 5

x-int: (0, 0)

y-int: (0, 0)

8) $k(x)=\dfrac{-3}{x^3+3x^2} = \dfrac{-3}{x^2(x+3)}$



Hole? no

VA: x = 0 , x = -3

HA or SA: y = 0

x-int: none

y-int: none

Sections 2.6 & 2.7 – I.C.E #2

9) $f(x)=\dfrac{x^2-3x-12}{x+2}$

$$\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-12)}}{2(1)}=\frac{3\pm\sqrt{57}}{2}$$

Hole? no

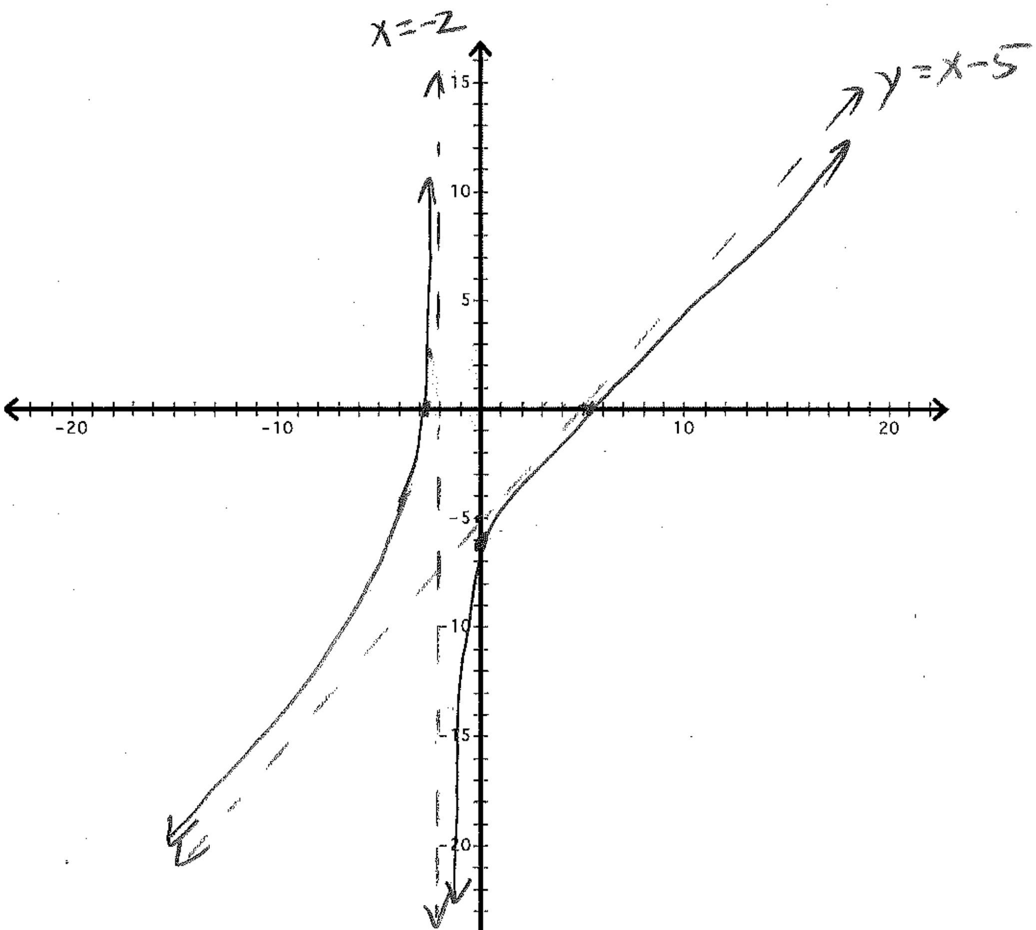
VA: $x=-2$

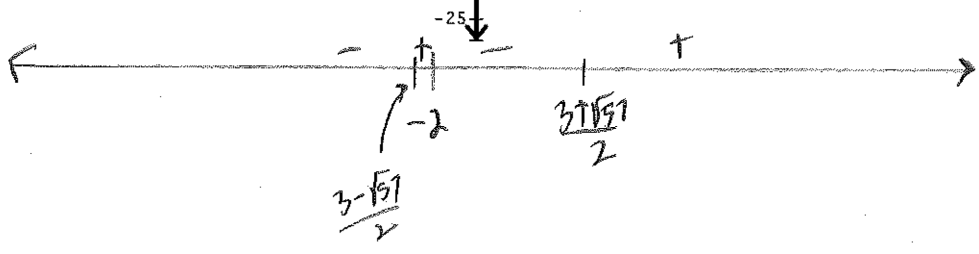
HA or SA: $y=x-5$

x-int: $\left(\frac{3+\sqrt{57}}{2},0\right),\left(\frac{3-\sqrt{57}}{2},0\right)$

y-int: $(0,-6)$

$$\begin{array}{r|rrr} -2 & 1 & -3 & -12 \\ & & -2 & 10 \\ \hline & 1 & -5 & -2 \end{array}$$





Sections 2.6 & 2.7 – I.C.E #2

10) $f(x)=\dfrac{3x^3-21x+18}{x^2-x-12}$ (hint: try x - 1 as a factor for the numerator)

$$\begin{array}{r|rrrr} 1 & 3 & 0 & -21 & 18 \\ & & 3 & 3 & -18 \\ \hline & 3 & 3 & -18 & 0 \end{array}$$

$(x-1)(3x^2+3x-18) = 3(x-1)(x^2+x-6)$

$= 3(x-1)(x+3)(x-2)$

$f(x) = \dfrac{3(x-1)(x+3)(x-2)}{(x-4)(x+3)}$

-60

Hole? yes: $x=-3$ $\left(-3, \frac{60}{7}\right)$

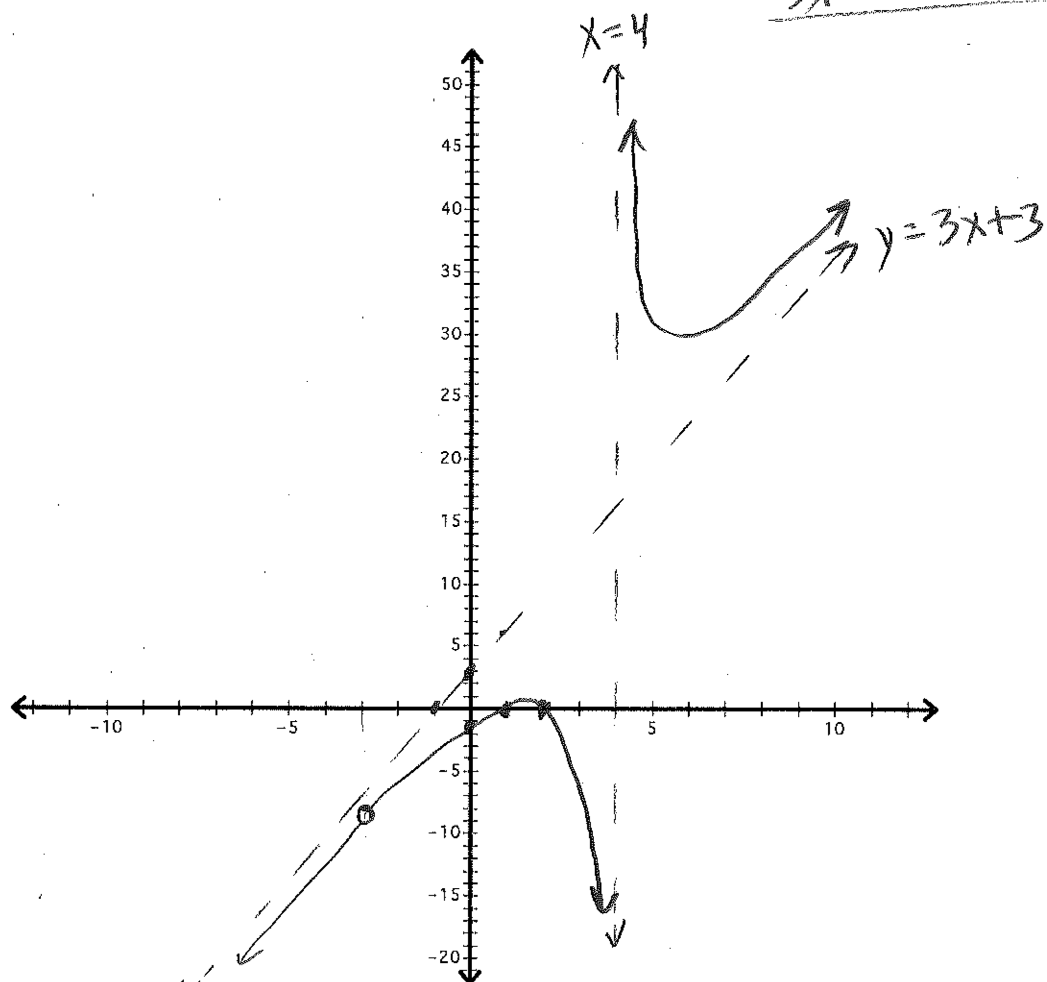
VA: $x=4$ (not $x=-3$ since hole)

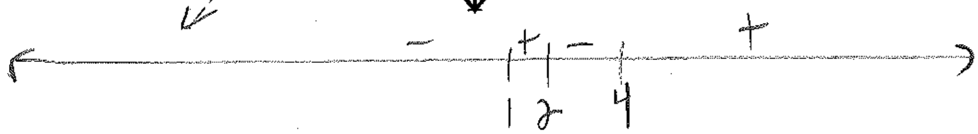
HA or SA: $y=3x+3$

x-int: $(1,0), (2,0)$

y-int: $(0, -3/2)$

$$\begin{array}{r|l} & 3x+3 \\ \hline x^2-x-12 & 3x^3-0x^2-21x+18 \\ & -(3x^3-3x^2-36x) \\ \hline & 3x^2+15x+18 \\ & 3x^2-3x-36 \\ \hline \end{array}$$





Sections 2.6 & 2.7 – I.C.E #2

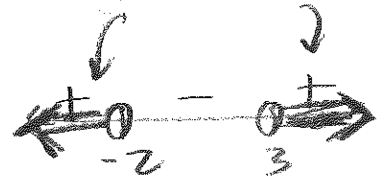
11) Solve $x^2 - x > 6$ and answer using interval notation

we want > 0

$x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$

critical #s $3, -2$

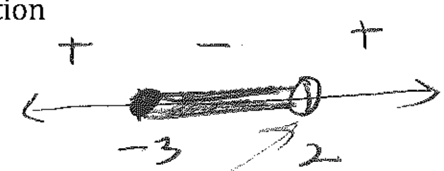


$\boxed{(-\infty, 2) \cup (3, \infty)}$

12) Solve $\dfrac{2(x+3)}{x-2} \le 0$ and answer using interval notation

critical #s $-3, 2$

$D: \{x \mid x \neq 2\}$



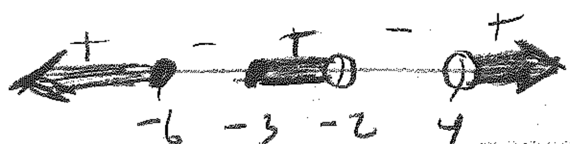
$\boxed{[-3, 2)}$

13) Solve $\dfrac{x^2+9x+18}{x^2-2x-8} \ge 0$ and answer using interval notation

$\dfrac{(x+3)(x+6)}{(x-4)(x+2)} \geq 0$

critical #s $-3, -6, 4, -2$

$D: \{x \mid x \neq 4, x \neq -2)$



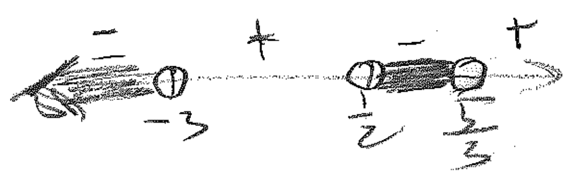
$\boxed{(-\infty, 6] \cup [3, -2) \cup (4, \infty)}$

14) Solve $\dfrac{4}{x+3} - \dfrac{2}{2x-1} < 0$ and answer using interval notation

$\dfrac{4(2x-1) - 2(x+3)}{(x+3)(2x-1)} = \dfrac{8x - 4 - 2x - 6}{(x+3)(2x-1)} = \dfrac{6x - 10}{(x+3)(2x-1)} = \dfrac{2(3x-5)}{(x+3)(2x-1)}$

critical #s $\frac{5}{3}, -3, \frac{1}{2}$

$D: \{x \mid x \neq -3, x \neq \frac{1}{2}\}$



$\boxed{(-\infty, -3) \cup \left(\frac{1}{2}, \frac{5}{3}\right)}$