

KEY

Sections 2.6 & 2.7 - I.C.E

Find all horizontal, slant, and vertical asymptotes for each function. Be sure to state your answers as equations of lines. Also find all x and y-intercepts and draw a sketch of the graph. Label where the asymptotes and the intercepts are located on your graph.

1) $f(x) = \frac{x}{x-3}$

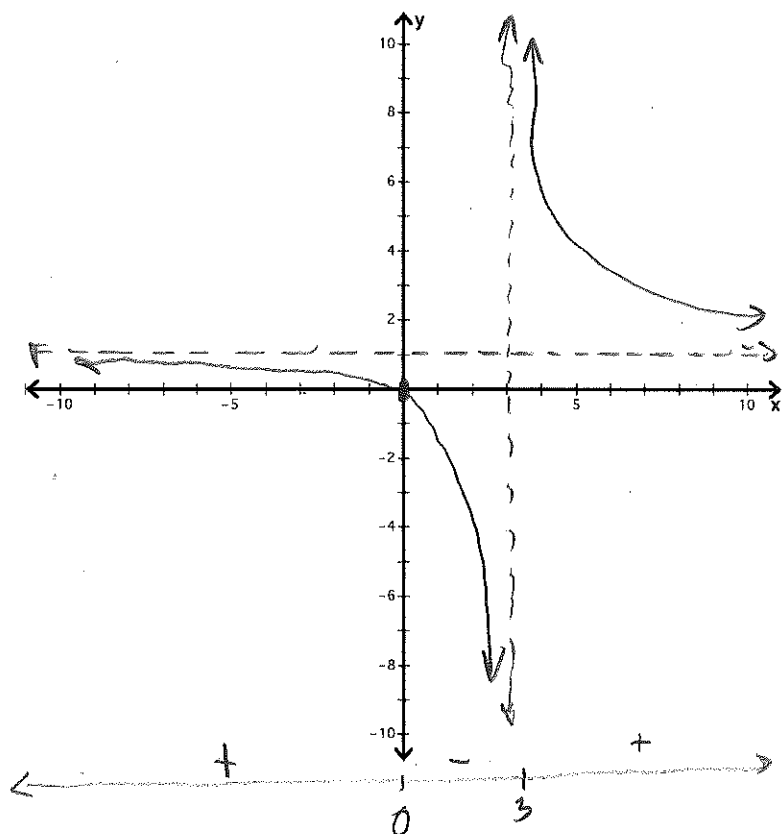
Hole? none

VA: $x=3$

HA or SA: $y=1$

x-int: $(0,0)$

y-int: $(0,0)$



2) $f(x) = \frac{-x^2}{x^2-16} = \frac{-x^2}{(x+4)(x-4)}$

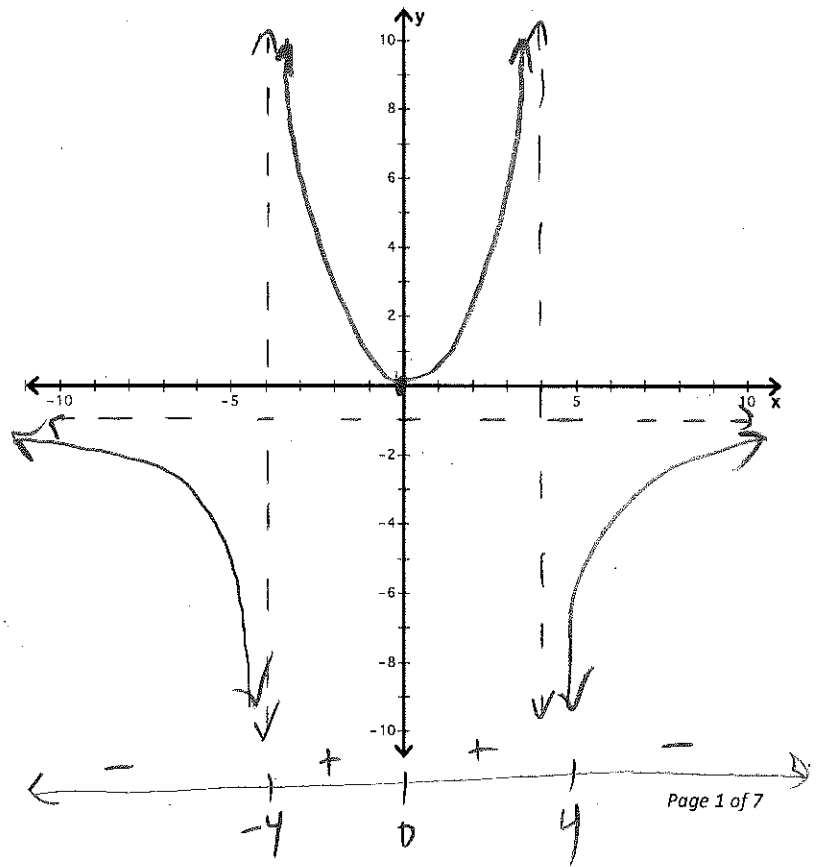
Hole? none

VA: $x=4, x=-4$

HA or SA: $y=-1$

x-int: $(0,0)$

y-int: $(0,0)$



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$$3) f(x) = \frac{x^2 + 4x + 3}{2x^2 - 2x - 4} = \frac{(x+3)(x+1)}{2(x-2)(x+1)}$$

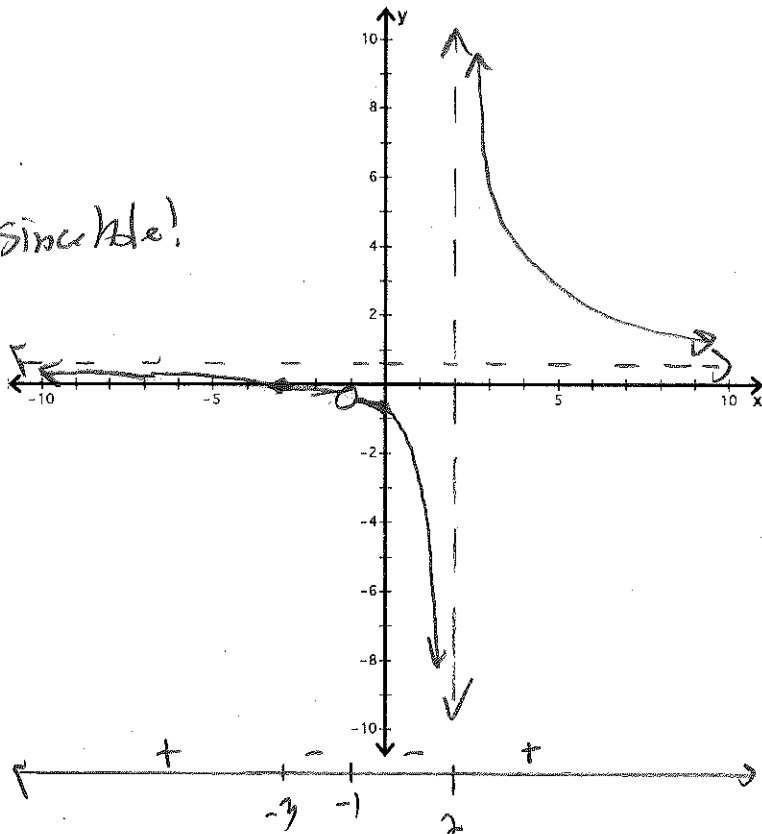
Hole? $x = -1$ $(-1, \frac{1}{2})$

VA: $x = 2$ not @ $x = -1$ since hole!

HA or SA: $y = \frac{1}{2}$

x-int: $(-3, 0)$

y-int: $(0, -\frac{3}{4})$



$$4) f(x) = \frac{3x+1}{x-2}$$

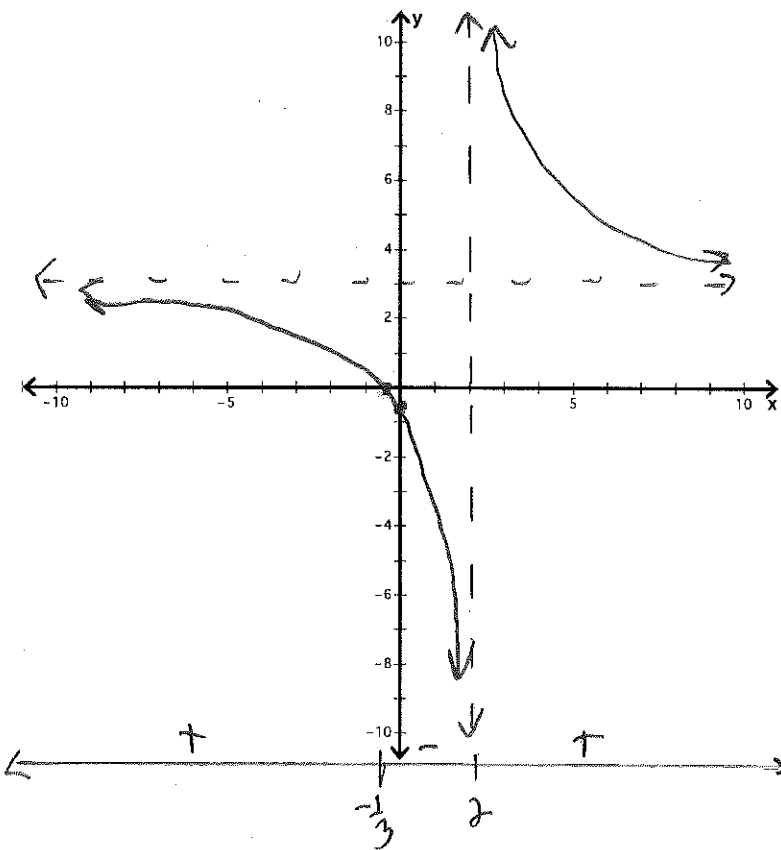
Hole? no

VA: $x = 2$

HA or SA: $y = 3$

x-int: $(-\frac{1}{3}, 0)$

y-int: $(0, -\frac{1}{2})$



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5) $g(x) = \frac{5}{x^2 - 9} = \frac{5}{(x+3)(x-3)}$

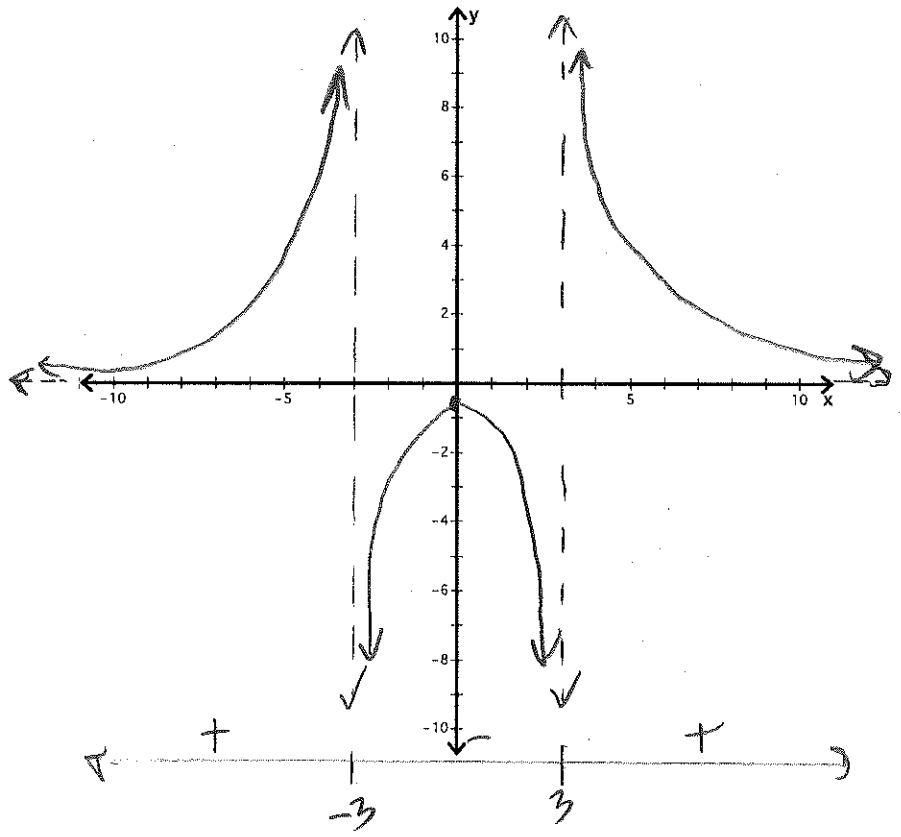
Hole? no

VA: $x=3, x=-3$

HA or SA: $y=0$

x-int: none

y-int: $(0, -5/9)$



6) $h(x) = \frac{2x}{x^2 - 1} = \frac{2x}{(x+1)(x-1)}$

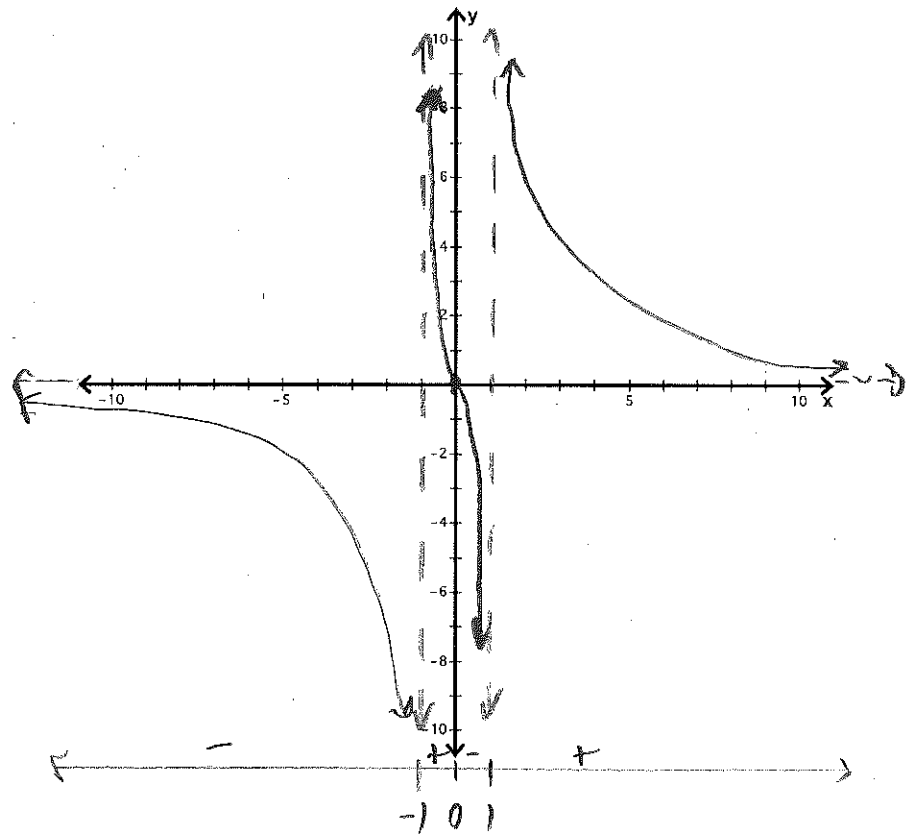
Hole? no

VA: $x=-1, x=1$

HA or SA: $y=0$

x-int: $(0, 0)$

y-int: $(0, 0)$

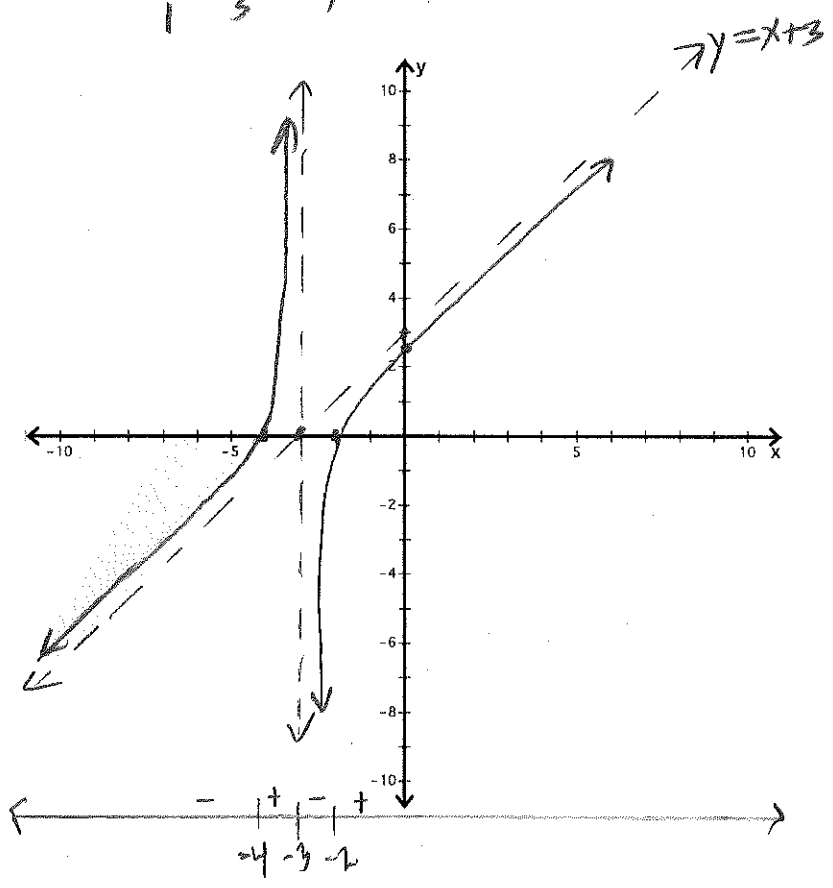


Sections 2.6 & 2.7 - I.C.E

$$\begin{array}{r} -3 \overline{) 1 \ 6 \ 8} \\ \underline{-3 \ -9} \\ 1 \ 3 \ -1 \end{array}$$

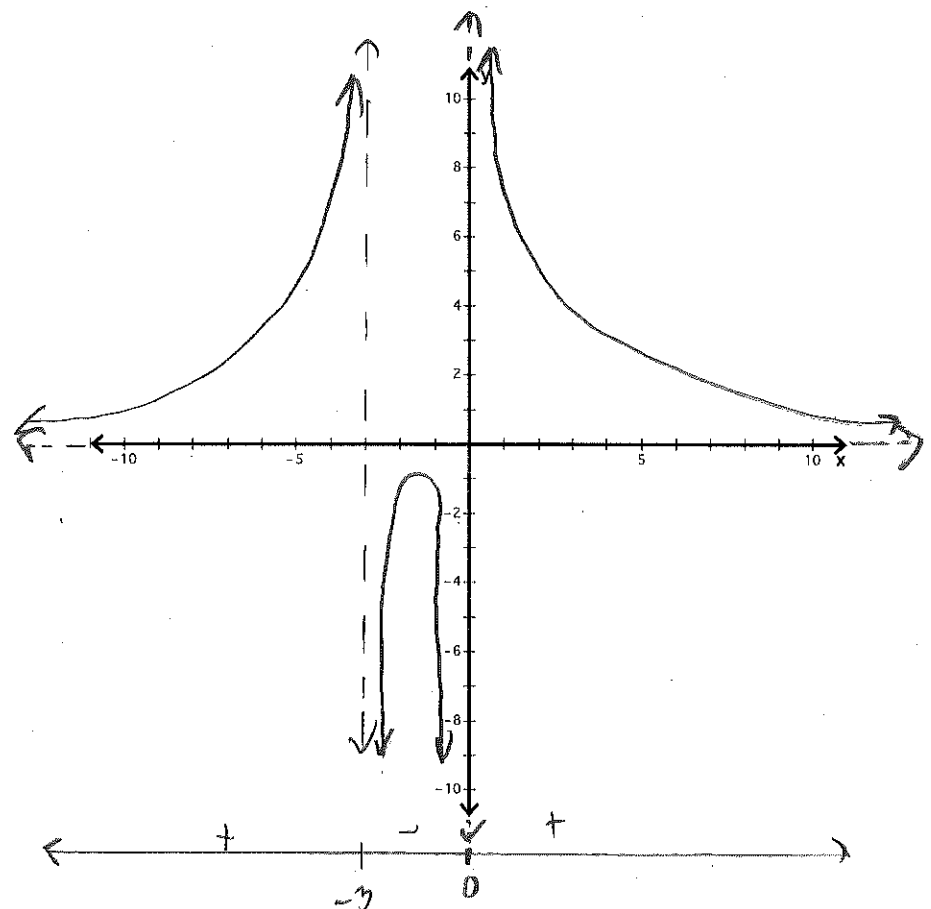
7) $f(x) = \frac{x^2 + 6x + 8}{x + 3} = \frac{(x+4)(x+2)}{x+3}$

Hole? NO
 VA: $x = -3$
 HA or SA: $y = x + 3$
 x-int: $(-4, 0)$ $(-2, 0)$
 y-int: $(0, 8/3)$



8) $k(x) = \frac{2}{x^2 + 3x} = \frac{2}{x(x+3)}$

Hole? NO
 VA: $x = 0, x = -3$
 HA or SA: $y = 0$
 x-int: none
 y-int: none



Sections 2.6 & 2.7 - I.C.E

9) $f(x) = \frac{x^2 + 2x - 12}{x - 5}$

not factorable → quadratic formula!

$$\frac{-2 \pm \sqrt{2^2 - 4(1)(-12)}}{2(1)}$$

$$= -1 \pm \frac{\sqrt{52}}{2} = -1 \pm \frac{2\sqrt{13}}{2} = -1 \pm \sqrt{13}$$

Hole? NO

VA: $x = 5$

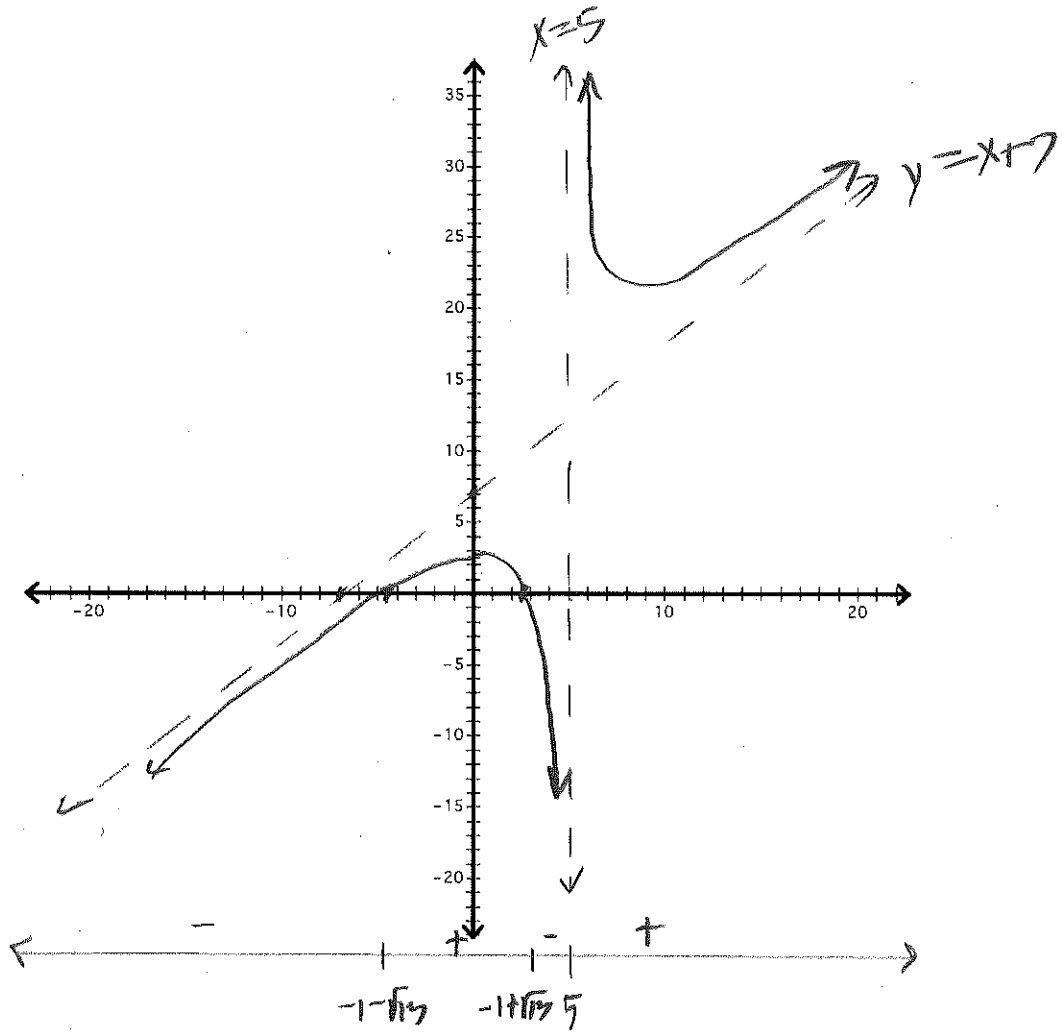
HA or SA: $y = x + 7$

x-int: $(-1 + \sqrt{13}, 0), (-1 - \sqrt{13}, 0)$

y-int: $(0, 12/5)$

$$\begin{array}{r|rrr} 5 & 1 & 2 & -12 \\ & & 5 & 35 \\ \hline & 1 & 7 & 23 \end{array}$$

ignore remainder



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$$\begin{array}{r} -1 \ 1 \ 0 \ -7 \ -6 \\ -1 \ 1 \ 6 \\ \hline 1 \ -1 \ -6 \ 0 \end{array}$$

10) $f(x) = \frac{4x^3 - 28x - 24}{x^2 - 3x - 10}$ (hint: try $x + 1$ as a factor for the numerator)

$$= 4(x^3 - 7x - 6) = 4(x+1)(x^2 - x - 6) = 4(x+1)(x-3)(x+2)$$

Hole? $x = -2$ $(-2, -24/9)$

VA: $x = 5$ (not $-2 \rightarrow$ hole!)

HA or SA: $y = 4x + 12$

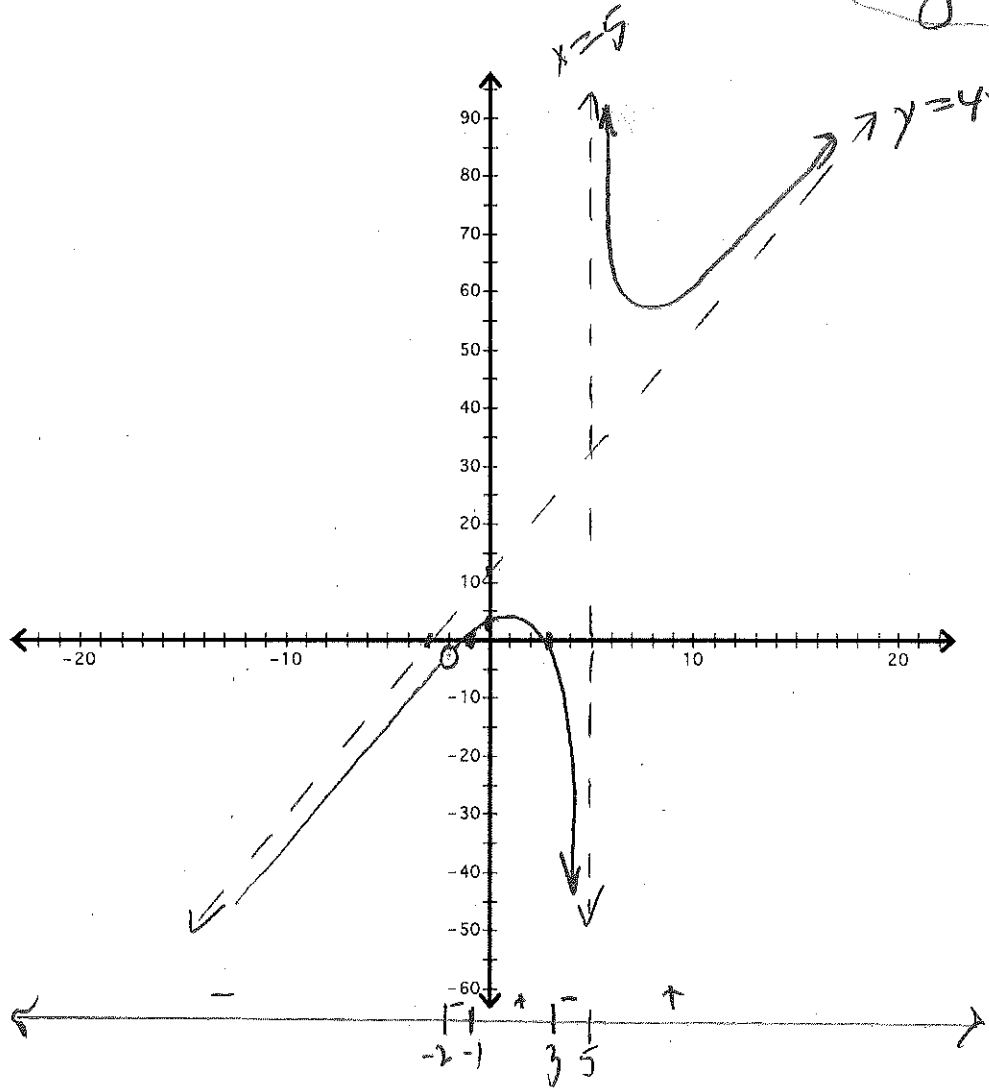
x-int: $(-1, 0), (3, 0)$ (not $(-2, 0) \rightarrow$ hole!)

y-int: $24/10 \rightarrow (0, 12/5)$

$$\frac{4(x+1)(x-3)(x+2)}{(x-5)(x+2)} \quad \frac{20}{-7}$$

$$\begin{array}{r} 4x + 12 \\ x^2 - 3x - 10 \overline{) 4x^3 - 0x^2 - 28x - 24} \\ -(4x^3 - 12x^2 - 40x) \\ \hline 12x^2 + 12x - 24 \\ -(12x^2 - 36x - 120) \\ \hline \text{ignore remainder} \end{array}$$

Slant A \rightarrow



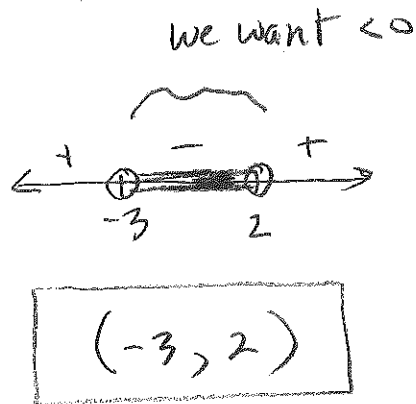
Sections 2.6 & 2.7 - I.C.E

11) Solve $x^2 + x < 6$ and answer using interval notation

$$x^2 + x - 6 < 0$$

$$(x+3)(x-2) < 0$$

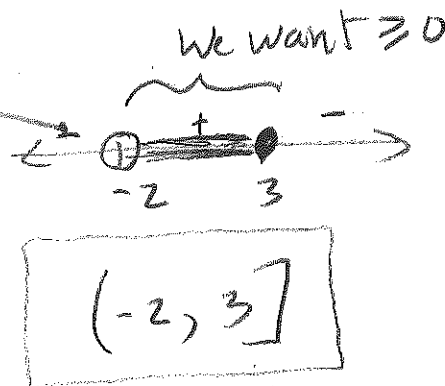
Critical values $-3, 2$



12) Solve $\frac{-2(x-3)}{x+2} \geq 0$ and answer using interval notation

D: $\{x \mid x \neq -2\}$

critical values $3, -2$

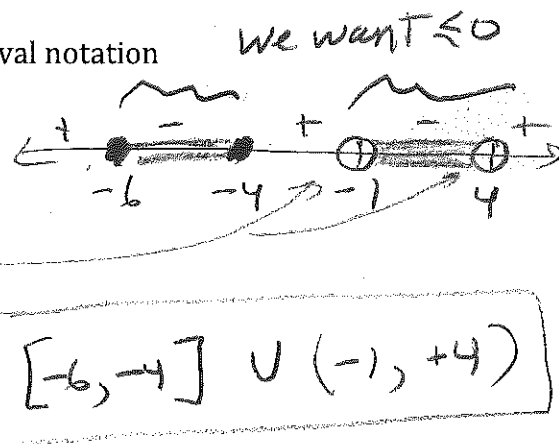


13) Solve $\frac{x^2 + 10x + 24}{x^2 - 3x - 4} \leq 0$ and answer using interval notation

$$\frac{(x+4)(x+6)}{(x-4)(x+1)} \leq 0$$

D: $\{x \mid x \neq 4, -1\}$

Critical #s $\rightarrow -4, -6, 4, -1$



14) Solve $\frac{4}{x+5} - \frac{1}{2x+3} > 0$ and answer using interval notation

$$\frac{4(2x+3) - 1(x+5)}{(x+5)(2x+3)} > 0$$

$$\frac{8x+12-x-5}{(x+5)(2x+3)} > 0$$

$$\frac{7x+7}{(x+5)(2x+3)} > 0$$

$$\frac{7(x+1)}{(x+5)(2x+3)} > 0$$

critical #s $\rightarrow -1, -5, -\frac{3}{2}$



$(-5, -\frac{3}{2}) \cup (-1, \infty)$