Pre-Calculus CP 1 – Section 3.5 Day 2 HW **Name:** KEY

Application Word Problems!

1. A deposit of $10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 5 years.

   a) What is the annual interest rate for this account?

$$2 = e^{r(5)}$$
$$\ln 2 = 5r$$
$$r = \frac{\ln 2}{5} \approx .138629 \Rightarrow 13.86\%$$

   b) Find the balance after 1 year.

$$A = 10{,}000e^{.1386(1)} \approx \$11{,}486.65$$

2. The half-life of radioactive uranium II is 250,000 years. What percent of a present amount of radioactive uranium II will remain after 5000 years?

$$\frac{1}{2} = e^{250{,}000k}$$
$$k = \frac{\ln(\frac{1}{2})}{250{,}000} \approx -.000002772588$$
$$A = e^{k(5000)} \approx .9862 \Rightarrow 98.62\%$$

3. The population of South Carolina (in thousands) from 1990 through 2003 can be modeled by $P(t)3499e^{0.0135t}$, where t is the time in years, with t = 0 corresponding to 1990. According to this model, when will the population reach 4.5 million?

$$4{,}500 = 3499e^{.0135t}$$
$$\frac{4500}{3499} = e^{.0135t}$$
$$t = \frac{\ln\left(\frac{4500}{3499}\right)}{.0135} \approx 18.64 \Rightarrow \boxed{2008}$$

Application Word Problems!

4. In a typing class, the average number N of words per minute typed after t weeks of lessons was found to be

$$N = \frac{157}{1+5.4e^{-0.12t}}$$

a) What is the carrying capacity for this problem?

157 words

b) Find the time necessary to type 50 words per minute

$$50 = \frac{157}{1+5.4e^{-.12t}}$$

$$5.4e^{-.12t} = \frac{157}{50} - \frac{50}{50} = \frac{107}{50}$$

$$e^{-.12t} = .3963$$

$$t = \frac{\ln(.3963)}{-.12} \approx 7.71 \text{ weeks}$$

5. The relationship between the number of decibels B and the intensity of a sound I in watts per square centimeter is $B = 10\log\left(\frac{I}{10^{-16}}\right)$. Determine the intensity of a sound in watts per square centimeter if the decibel level is 125.

$$125 = 10\log\left(\frac{I}{10^{-16}}\right)$$

$$12.5 = \log_{10}\left(\frac{I}{10^{-16}}\right)$$

$$10^{12.5} = \frac{I}{10^{-16}}$$

$$I = \frac{10^{12.5}}{10^{16}} = 10^{-3.5} \text{ w/cm}^2$$

Application Word Problems!

6. On a day a person is born, a deposit of $50,000 is made in a trust fund that pays 8.75% interest, compounded continuously.

   a) Find the balance on the person's 35th birthday.

$50{,}000e^{.0875(35)} \approx \$1{,}069{,}047.14$

   b) How much longer would the person have to wait for the balance in the trust fund to double?

$2138094.28 = 50000e^{.0875t}$

$42.76 = e^{.0875t}$

$\frac{\ln(42.76)}{.0875} = t \approx 42.92 \Rightarrow 43$

$43 - 35 = 8$ more years

7. Let Q represent a mass of plutonium 241 in grams, whose half-life is 14.4 years. The quantity of plutonium 241 present after t years is given by $Q = 100\left(\frac{1}{2}\right)^{t/14.4}$

   a) Determine the initial quantity

$Q = 100\left(\frac{1}{2}\right)^{0} =$ 100 gms

   b) Determine the quantity present after 10 years.

$100\left(\frac{1}{2}\right)^{10/14.4} \approx$ 61.795 grams

Application Word Problems!

8. The antler spread **a** (in inches) and shoulder height **h** (in inches) of an adult male American elk are related by the model $h = 116\log(a+40) - 176$. Approximate the shoulder height of a male American elk with an antler spread of 55 inches.

$h = 116\log(55+40) - 176$
$= 116\log(95) - 176 \approx$ 53.42 in

9. Detectives respond to a call at Dunkin Donuts made by Dana at exactly 5:10 AM. When they arrive Dana is panicked and visually upset. He says he arrived at work around 5 AM to open the store with a fellow worker. Dana has a witness in his father who says he dropped him off at exactly 5 AM. Dana tells the police that when he entered the store his fellow co-worker was already dead. The coroner arrives to take some temperatures and finds that the body is 85°F and the room it's in is kept at a constant 68°F. These temperatures are taken at exactly 5:55 AM. Two hours later the coroner takes the second temperature reading. He finds the body to be 74°F and the room to still be 68°F. Should the police consider Dana a suspect?

$u(t) = T + (u_0 - T)e^{kt}$ $\quad T = 68$ $\quad u_0 = 85$ $\quad t = 2$ $\quad u(2) = 74$

$74 = 68 + (85-68)e^{k(2)}$
$6 = 17e^{2k}$
$\ln\left(\frac{6}{17}\right) = 2k = k = -.52073$

$85 = 68 + (98.6-68)e^{-.52t}$
$\frac{17}{30.6} = e^{-.52t} \Rightarrow$ $t = 1.13$ hrs $= 42$ min

Dana is not a suspect!

10. Suppose a body is 83°F at 10 PM and that the air temperature around it is ~~55~~°F. ~~The body is quickly moved to a different room where the air temperature is 42°F~~. After one hour the body is found to be 76°F. Estimate the time of death.

$76 = 42 + (83-42)e^{k(1)}$
$34 = 41e^{k}$
$\ln\left(\frac{34}{41}\right) = k \approx -.1872$

$83 = 42 + (98.6-42)e^{-.1872t}$
$41 = 56.6e^{-.1872t}$

$\Rightarrow t = \frac{\ln\left(\frac{41}{56.6}\right)}{-.1872}$
$t = 1.72$ hrs
$\therefore$ 8:17 PM