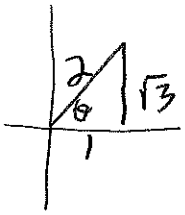
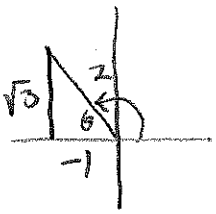
Sections 4.7-4.8 – I.C.E.

For 1-9, evaluate without using a calculator. Give answer in both degrees and radians.

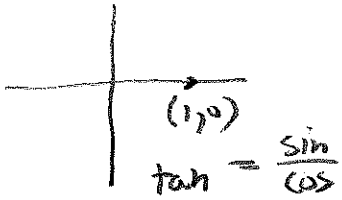
1. $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$



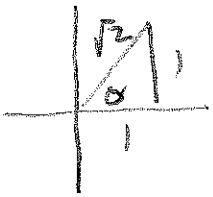
2. $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



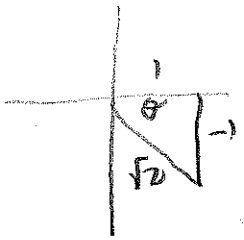
3. $\arctan(0) = 0$



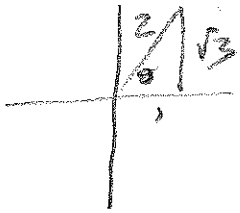
4. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$



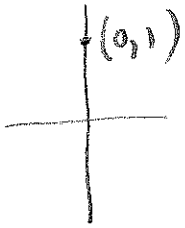
5. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$



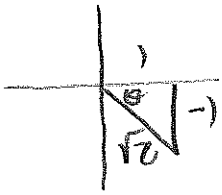
6. $\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$



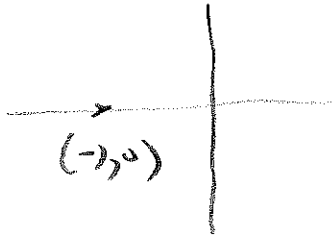
7. $\arcsin(1) = \frac{\pi}{2}$



8. $\arctan(-1) = -\frac{\pi}{4}$

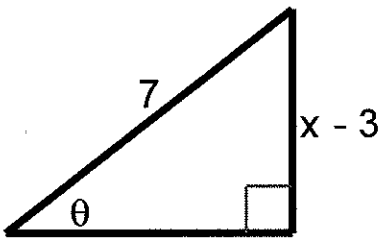


9. $\cos^{-1}(-1) = \pi$



Sections 4.7-4.8 – I.C.E.

10. Use an inverse function to write θ as a function of x.



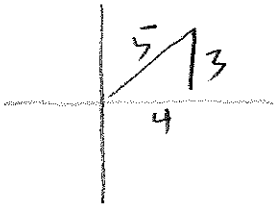
$\sin\theta = \frac{x-3}{7}$

$\theta = \arcsin\left(\frac{x-3}{7}\right)$

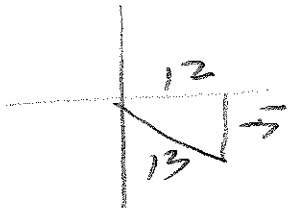
$\theta = \arcsin\left(\frac{x-3}{7}\right)$

For 11-14, find the exact value of the expression without using a calculator.

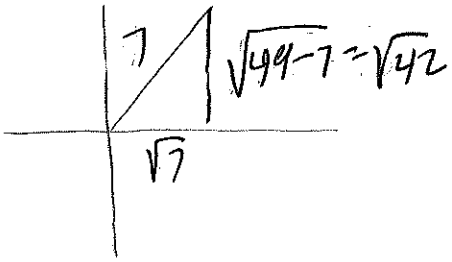
11. $\cos\left(\arctan\left(\frac{3}{4}\right)\right) = \frac{4}{5}$



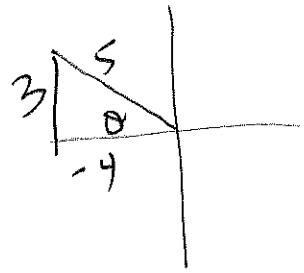
12. $\sec\left(\sin^{-1}\left(-\frac{5}{13}\right)\right) = \frac{13}{12}$



13. $\sin\left(\cos^{-1}\left(\frac{\sqrt{7}}{7}\right)\right) = \frac{\sqrt{42}}{7}$



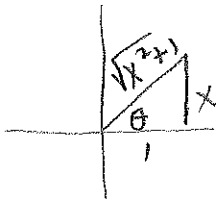
14. $\cot\left(\arccos\left(-\frac{4}{5}\right)\right) = -\frac{4}{3}$



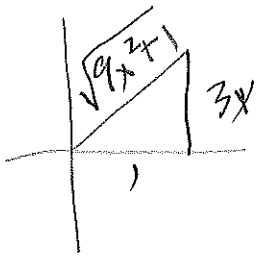
Sections 4.7-4.8 – I.C.E.

For 15-18, write an algebraic expression that is equivalent to the given expression (no calculator!!).

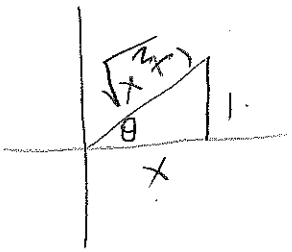
15. $\sin(\arctan(x)) = \frac{x}{\sqrt{x^2+1}} = \frac{x\sqrt{x^2+1}}{x^2+1}$



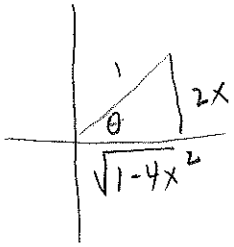
16. $\sec(\arctan(3x)) = \sqrt{9x^2+1}$



17. $\cot\left(\arctan\left(\frac{1}{x}\right)\right) = x$

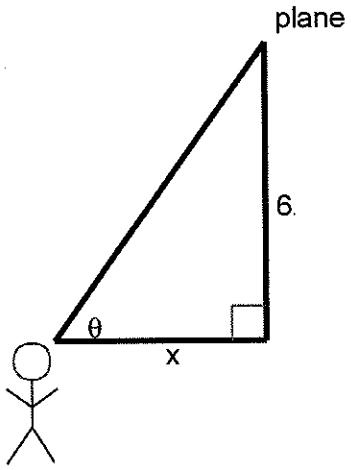


18. $\sec(\arcsin(2x)) = \frac{1}{\sqrt{1-4x^2}} = \frac{\sqrt{1-4x^2}}{1-4x^2}$



For questions 19-26, you can use a calculator!!

19. An airplane flies at an altitude of 6 miles toward a point directly over an observer. Drawing is not to scale



a) Write θ as a function of x.

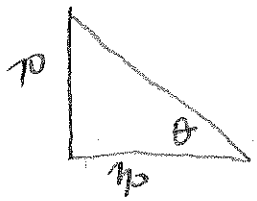
θ = $\arctan\left(\frac{6}{x}\right)$

b) Find θ when x = 7 miles.

θ = $\arctan\left(\frac{6}{7}\right) \approx 40.6°$

Sections 4.7-4.8 – I.C.E.

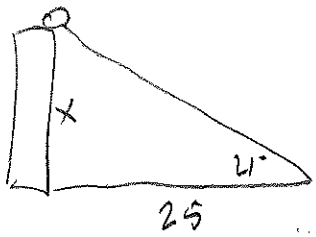
20. The height of a radio transmission tower is 70 meters, and it casts a shadow of length 30 meters. Find the angle of elevation the sun.



$\tan\theta = \frac{70}{30}$

$\theta = \arctan\left(\frac{70}{30}\right) \approx 66.8°$

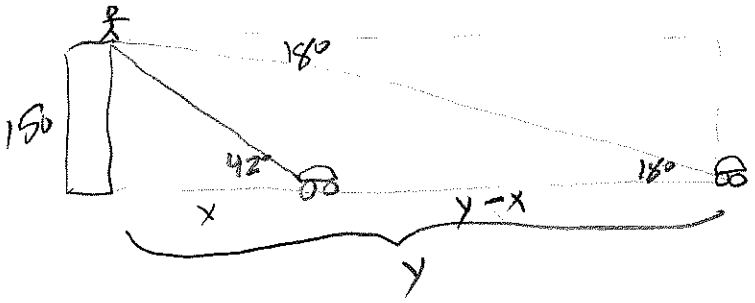
21. Your football has landed at the edge of the roof of your school building. When you are 25 feet from the base of the building, the angle of elevation to your football is 21°. How high off the ground is your football?



$\tan 21° = \frac{x}{25}$

$x = 25\tan 21° \approx 9.6 \text{ ft}$

22. From the top of a 150 ft. building Persie observes a car moving toward her. If the angle of depression of the car changes from 18° to 42° during the observation, how far does the car travel?



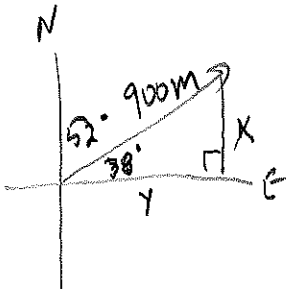
$\tan 18° = \frac{150}{y} \Rightarrow y = \frac{150}{\tan 18°}$

$\tan 42° = \frac{150}{x} \Rightarrow x = \frac{150}{\tan 42°}$

$\text{dist} = y - x = \frac{150}{\tan 18°} - \frac{150}{\tan 42°}$

$\approx 295.1 \text{ ft}$

23. An airplane flying at 600 mph has a bearing of 52°. After flying for 1.5 hours, how far north and how far east will the plane have traveled from its point of departure?

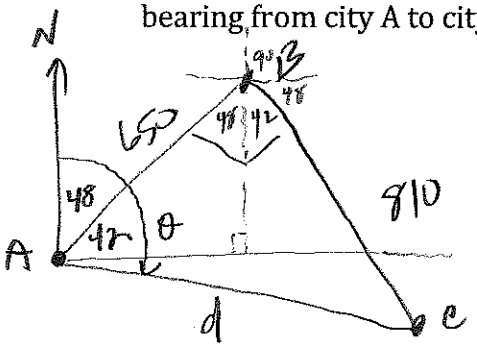


$\sin 38° = \frac{x}{900} \Rightarrow x = 900\sin 38° \approx 554.1 \text{ miles N}$

$\cos 38° = \frac{y}{900} \Rightarrow y = 900\cos 38° \approx 709.2 \text{ miles E}$

Sections 4.7-4.8 – I.C.E.

24. From city A to city B, a plane flies 650 miles at a bearing of 48°. From city B to city C, the plane flies 810 miles at a bearing of 138° degrees. Find the distance from city A to city C and the bearing from city A to city C.



$d^2 = 650^2 + 810^2 = 1114500$

$d = \sqrt{1114500} \approx 1070 \text{ m}$

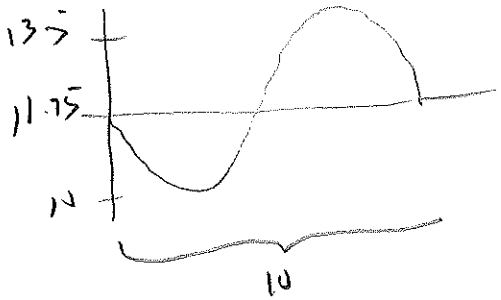
$\tan\theta = \frac{810}{650} \Rightarrow \theta = \arctan\frac{810}{650} \approx 51.25°$

Bearing $\approx 48 + 51.25 \approx 99.25°$

25. A buoy oscillates in simple harmonic motion as waves go past. It is noted that the buoy moves a total of 3.5 feet from its low point to its high point. It returns to its highest point every 10 seconds. At time = 0, the buoy is at its middle height getting lower. The lowest height is 10 feet.

a) What is the equation for the height of the buoy?

$y = -1.75\sin\frac{\pi x}{5} + 11.75$



$P = \frac{2\pi}{\omega} = 10$

$\omega = \frac{2\pi}{10} = \frac{\pi}{5}$

b) In your equation, what does the x-variable represent?

time in seconds

c) In your equation, what does the y-variable represent?

height

d) Predict the height of the buoy at 5 minutes and 15 seconds.

$y = -1.25\sin\frac{\pi}{5}(315) + 11.75 \approx 12.29 \text{ ft}$

Sections 4.7-4.8 – I.C.E.

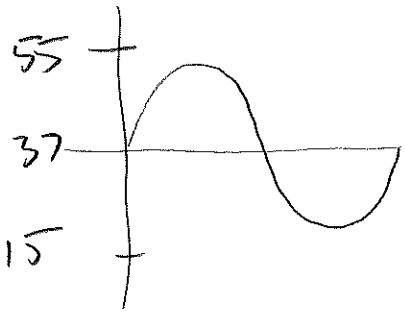
26. The Robotics team has gathered around a spring that has a weight attached to the end, and they are testing while it bobs up and down. Joey measures that 22 cm. is the maximum distance the weight moves vertically upward OR downward away from its equilibrium position. Sam observes that it takes the weight 3 seconds to complete one full cycle.

a) Assuming the weight starts at its equilibrium position of 37 inches above a table, write an equation for simple harmonic motion of the bottle (think sine or cosine curve).

$P = 3 = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{3}$

$y = 22 \sin \frac{2\pi x}{3} + 37$



b) What is the frequency of this harmonic motion?

$F = \frac{\omega}{2\pi} = \frac{2\pi/3}{2\pi} = \frac{2\pi}{6\pi} = \frac{1}{3}$