Sections 5.1 & 5.2 – I.C.E – Trig Identities

1) By starting with $\sin^2\theta+\cos^2\theta=1$, derive the other two Pythagorean Identities.

$$\frac{\sin^2\theta + \cos^2\theta = 1}{\sin^2} \qquad \frac{\sin^2\theta + \cos^2\theta = 1}{\cos^2\theta}$$

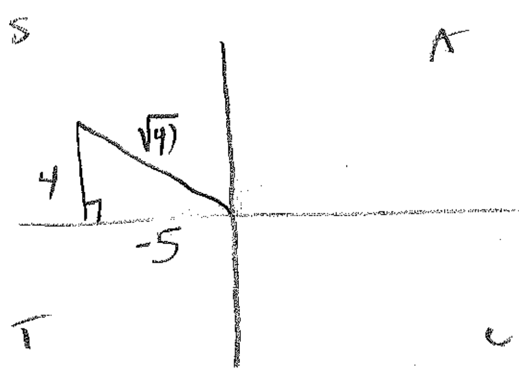
$$\Rightarrow 1 + \cot^2\theta = \csc^2\theta \qquad \Rightarrow \tan^2\theta + 1 = \sec^2\theta$$

2) If $\tan\theta=-\frac{4}{5}$ and $\sin\theta>0$, use the fundamental identities to find the other five trig functions for θ.

$\sin\theta = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$ $\qquad$ $\csc\theta = \frac{\sqrt{41}}{4}$

$\cos\theta = \frac{-5}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$ $\qquad$ $\sec\theta = -\frac{\sqrt{41}}{5}$

$\tan\theta = -\frac{4}{5}$ $\qquad$ $\cot\theta = -\frac{5}{4}$



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Simplify the following to ONE trig function or numerical value:

3) $\cos^2\beta+\cos^2\left(\frac{\pi}{2}-\beta\right)$

$=\cos^2\beta+\sin^2\beta$

$\boxed{=1}$

4) $\sin t\csc\left(\frac{\pi}{2}-t\right)$

$=\sin t\sec t$

$=\sin t\cdot\frac{1}{\cos t}=\boxed{\tan t}$

5) $\dfrac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$

$=\dfrac{\sin x}{\cos x}=\boxed{\tan x}$

6) $\sec y\cos y$

$=\left(\frac{1}{\cos y}\right)\cos y=\boxed{1}$

7) $(1+\sin y)(1+\sin(-y))$

$=(1+\sin y)(1-\sin y)$

$=1-\sin^2 y$

$=\boxed{\cos^2 y}$

8) $\sec^2\left(\frac{\pi}{2}-x\right)-1$

$=\csc^2 x-1$

$=\boxed{\cot^2 x}$

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Prove the following identities: be sure to only work ONE side of the equation!

9) $\dfrac{1}{\tan x}+\dfrac{1}{\cot x}=\tan x+\cot x$

$\dfrac{\cot x+\tan x}{\tan x \cdot \cot x} = \dfrac{\cot x+\tan x}{\tan x\left(\frac{1}{\tan x}\right)} = \cot x+\tan x$ ✓

10) $\cot\alpha+\tan\alpha=\csc\alpha\sec\alpha$

$\dfrac{1}{\tan\alpha}+\tan\alpha = \dfrac{1+\tan^2\alpha}{\tan\alpha} = \dfrac{\sec^2\alpha}{\tan\alpha}$

$= \dfrac{\sec^2\alpha}{\frac{\sin\alpha}{\cos\alpha}} = \dfrac{\sec^2\alpha\cos\alpha}{\sin\alpha} = \dfrac{\sec^2\alpha\csc\alpha}{\sec\alpha} = \boxed{\sec\alpha\csc\alpha}$ ✓

11) $\sin^2\alpha-\sin^4\alpha=\cos^2\alpha-\cos^4\alpha$

$\sin^2\alpha(1-\sin^2\alpha) = (1-\cos^2\alpha)\cos^2\alpha = \cos^2\alpha-\cos^4\alpha$ ✓

12) $\dfrac{\cot^3 t}{\csc t}=\cos t(\csc^2 t-1)$

$\cot^2 t\cot t\sin t = (\csc^2 t-1)\left(\dfrac{\cos t}{\cancel{\sin t}}\right)\dfrac{\cancel{\sin t}}{1}$

$= (\csc^2 t-1)\cos t$ ✓

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13) $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=2\sec\theta$

$$\frac{(1+\sin\theta)(1+\sin\theta)+\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{1+2\sin\theta+\overbrace{\sin^2\theta+\cos^2\theta}^{1}}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \underline{2\sec\theta} \checkmark$$

14) $\tan^2\theta+4=\sec^2\theta+3$

$$=(\sec^2\theta-1)+4 = \underline{\sec^2\theta+3} \checkmark$$

15) $\cot^2 y(\sec^2 y-1)=1$

$$\cot^2 y(\tan^2 y) = \left(\frac{1}{\tan^2 y}\right)\left(\frac{\tan^2 y}{1}\right) = \underline{1} \checkmark$$

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16) $\dfrac{1}{\sec x \tan x} = \csc x - \sin x$

$$\frac{1}{\sec x} \cdot \frac{1}{\tan x} = \cos x \cot x = \cos x \left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{\cos^2 x}{\sin x} = \frac{1-\sin^2 x}{\sin x} = \frac{1}{\sin x} - \frac{\sin^2 x}{\sin}$$

$$= \csc x - \sin x \checkmark$$

**work on the left side on the following problem (even though the right looks more complicated)

17) $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

$\checkmark$

$$\frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}} = \frac{\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}}$$

$$= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} \cdot \frac{\cos x \cos y}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y + \sin x \sin y} \checkmark$$