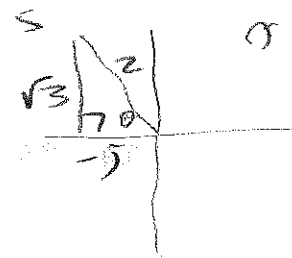


85.4 p 404-405 R 1, 7, 9, 15-18, 23-26, 31, 32, 37, 39, 41, 56, 63

(1) ^(a) $\cos(20+45) = \cos 20 \cos 45 - \sin 20 \sin 45$
 $= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$
 $= \frac{-1 - \sqrt{3}}{2\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{-\sqrt{2} - \sqrt{6}}{4}$



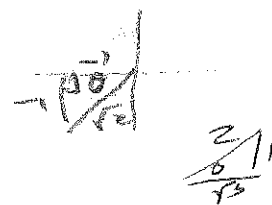
(b) $\cos 120 + \cos 45 = -\frac{1}{2} + \frac{1}{\sqrt{2}} = \frac{-\sqrt{2} + 2}{2\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{-2 + 2\sqrt{2}}{4} = \frac{\sqrt{2} - 1}{2}$

(7) $\tan 105 = \tan(60+45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$
 $= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \left(\frac{1 + \sqrt{3}}{1 + \sqrt{3}} \right) = \frac{1 + 2\sqrt{3} + 3}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2} = -(2 + \sqrt{3}) = -2 - \sqrt{3}$

$\sin 105 = \sin(60+45) = \sin 60 \cos 45 + \cos 60 \sin 45$
 $= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right) = \frac{\sqrt{3} + 1}{2\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{4}$

$\cos 105 = \cos(60+45) = \cos 60 \cos 45 - \sin 60 \sin 45$
 $= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1 - \sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2} - \sqrt{6}}{4}$

(9) $\sin 195 = \sin(225 - 30)$
 $= \sin 225 \cos 30 - \cos 225 \sin 30$
 $= -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \left(-\frac{1}{\sqrt{2}} \cdot \frac{1}{2} \right) = \frac{-\sqrt{6} + \sqrt{2}}{4}$



$\cos 195 = \cos(225 - 30) = \cos 225 \cos 30 + \sin 225 \sin 30$
 $= -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{1}{\sqrt{2}} \cdot \frac{1}{2} \right) = \frac{-\sqrt{6} - \sqrt{2}}{4}$

$\tan 195 = \tan(225 - 30) = \frac{\tan 225 - \tan 30}{1 + \tan 225 \tan 30}$

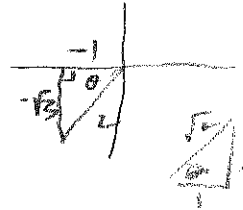
$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \left(\frac{1}{\sqrt{3}} \right)} = \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \cdot \frac{3 - 2\sqrt{3} + 1}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$

15

$$\sin 285 = \sin(240 + 45)$$

$$= \sin 240 \cos 45 + \cos 240 \sin 45$$

$$= \frac{-\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{-\sqrt{3} - 1}{2\sqrt{2}} = \frac{-\sqrt{6} - \sqrt{2}}{4}$$



$$\cos 285 = \cos(240 + 45) = \cos 240 \cos 45 - \sin 240 \sin 45$$

$$= -\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{-\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{-1 + \sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\tan 285 = \tan(240 + 45) = \frac{\tan 240 + \tan 45}{1 - \tan 240 \tan 45}$$

$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} = \frac{\sqrt{3} + 1 + (\sqrt{3})}{1 - \sqrt{3}(1 + \sqrt{3})} = \frac{3 + 2\sqrt{3} + 1}{1 - 3}$$

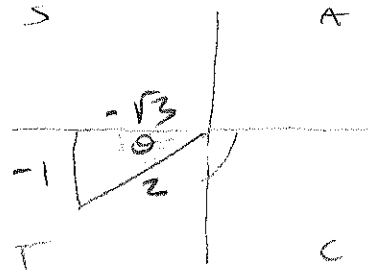
$$= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

16 $\sin(-105) = \sin 255 = \sin(210 + 45)$

$$= \sin 210 \cos 45 + \cos 210 \sin 45$$

$$= -\frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{-\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{-1 - \sqrt{3}}{2\sqrt{2}} = \frac{-\sqrt{2} - \sqrt{6}}{4}$$



$$\cos(-105) = \cos(210 + 45)$$

$$= \cos 210 \cos 45 - \sin 210 \sin 45$$

$$= -\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{-\sqrt{3} - 1}{2\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

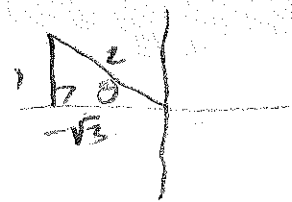
$$\tan(-105) = \tan(210 + 45)$$

$$= \frac{\tan 210 + \tan 45}{1 - \tan 210 \tan 45} = \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1} = \frac{1 + \sqrt{3}}{\sqrt{3} - 1}$$

$$= \frac{1 + \sqrt{3}(-1 - \sqrt{3})}{-1 + \sqrt{3}(-1 - \sqrt{3})} = \frac{-1 - 2\sqrt{3} - 3}{-1 - 3} = \frac{-4 - 2\sqrt{3}}{-2}$$

$$= 2 + \sqrt{3}$$

$$\begin{aligned}
 \textcircled{17} \quad \sin(-165) &= \sin(195) = \sin(150+45) \\
 &= \sin 150 \cos 45 + \cos 150 \sin 45 \\
 &= \frac{1}{2} \left(\frac{\sqrt{2}}{2} \right) + -\frac{\sqrt{3}}{2} \left(\frac{\sqrt{2}}{2} \right) \\
 &= \frac{\sqrt{2} - \sqrt{6}}{4}
 \end{aligned}$$



$$\begin{aligned}
 \cos(-165) &= \cos(150+45) = \cos 150 \cos 45 - \sin 150 \sin 45 \\
 &= -\frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{1}{2} \frac{\sqrt{2}}{2} = \frac{-\sqrt{6} - \sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \tan(-165) &= \tan(150+45) = \frac{\tan 150 + \tan 45}{1 - \tan 150 \tan 45} \\
 &= \frac{-\frac{1}{\sqrt{3}} + 1}{1 - (-\frac{1}{\sqrt{3}})} = \frac{-\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}+1} (\sqrt{3}-1) = \frac{3-2\sqrt{3}+1}{3-1} \\
 &= \frac{4-2\sqrt{3}}{2} = \boxed{2-\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{18} \quad \sin 15^\circ &= \sin(45^\circ - 30^\circ) = \sin 45 \cos 30 - \cos 45 \sin 30 \\
 &= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \frac{1}{2} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}
 \end{aligned}$$

$$\begin{aligned}
 \cos 15^\circ &= \cos(45^\circ - 30^\circ) = \cos 45 \cos 30 + \sin 45 \sin 30 \\
 &= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}
 \end{aligned}$$



$$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \left(\frac{\sqrt{3}}{3} \right)} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{9 - 6\sqrt{3} + 3}{9 - 3} = \frac{12 - 6\sqrt{3}}{6} = \boxed{2 - \sqrt{3}}$$

$$\textcircled{23} \cos 25^\circ \cos 15^\circ - \sin 25^\circ \sin 15^\circ = \cos(25+15) = \cos 40^\circ$$

$$\textcircled{24} \sin 140^\circ \cos 50^\circ + \cos 140^\circ \sin 50^\circ = \sin(140+50) = \sin 190^\circ$$

$$\textcircled{25} \frac{\tan 325^\circ - \tan 86^\circ}{1 + \tan 325^\circ \tan 86^\circ} = \tan(325-86) = \tan 239^\circ$$

$$\textcircled{26} \frac{\tan 140^\circ - \tan 60^\circ}{1 + \tan 140^\circ \tan 60^\circ} = \tan(140-60) = \tan 80^\circ$$

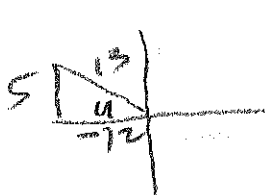
$$\textcircled{31} \sin 330^\circ \cos 30^\circ - \cos 330^\circ \sin 30^\circ = \sin(330-30) = \sin 300^\circ = -\frac{\sqrt{3}}{2}$$



$$\textcircled{32} \cos 15^\circ \cos 60^\circ + \sin 15^\circ \sin 60^\circ = \cos(15-60) = \cos(-45) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



$$\textcircled{37} \sin u = \frac{5}{13} \quad \cos v = -\frac{3}{5} \quad \text{II}$$



$$\begin{aligned} \sin(u+v) &= \sin u \cos v + \cos u \sin v \\ &= \frac{5}{13} \cdot -\frac{3}{5} + -\frac{12}{13} \cdot \frac{4}{5} = \frac{-15-48}{65} = -\frac{63}{65} \end{aligned}$$

$$\textcircled{39} \cos(u+v) = \cos u \cos v - \sin u \sin v = -\frac{12}{13} \cdot -\frac{3}{5} - \frac{5}{13} \cdot \frac{4}{5} = \frac{36-20}{65} = \frac{16}{65}$$

$$\textcircled{41} \tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{-\frac{5}{12} + -\frac{4}{3}}{1 - (-\frac{5}{12})(-\frac{4}{3})} = \frac{-15-48}{36} = \frac{36-20}{36} = \frac{-63}{16}$$

$$\textcircled{56} \sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\sin\frac{\pi}{2} \cos x + \cos\frac{\pi}{2} \sin x$$

$$1 \cos x + 0 \cdot \sin x = \cos x \checkmark$$

(91)

$$\textcircled{63} \sin(x+y) + \sin(x-y) = 2\sin x \cos y$$

$$= (\sin x \cos y + \cos x \sin y) + (\sin x \cos y - \cos x \sin y)$$

$$= 2\sin x \cos y \checkmark$$