

Section 5.4 – Review of Sum and Difference Formulas

$$\sin(\theta + \beta) = \sin\theta \cos\beta + \cos\theta \sin\beta$$

$$\sin(\theta - \beta) = \sin\theta \cos\beta - \cos\theta \sin\beta$$

$$\cos(\theta + \beta) = \cos\theta \cos\beta - \sin\theta \sin\beta$$

$$\cos(\theta - \beta) = \cos\theta \cos\beta + \sin\theta \sin\beta$$

$$\tan(\theta + \beta) = \frac{\tan\theta + \tan\beta}{1 - \tan\theta \tan\beta}$$

$$\tan(\theta - \beta) = \frac{\tan\theta - \tan\beta}{1 + \tan\theta \tan\beta}$$



Find the EXACT value of the following using sum and difference formulas:

1) $\sin(15^\circ) = \sin(45^\circ - 30^\circ)$

$$\begin{aligned} &= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

2) $\tan(75^\circ) = \tan(45^\circ + 30^\circ)$

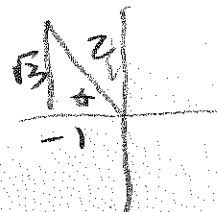
$$\begin{aligned} &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \cdot \left(\frac{\sqrt{3}}{3}\right)} \\ &= \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{(3 + \sqrt{3})}{(3 + \sqrt{3})} \\ &= \frac{9 + 6\sqrt{3} + 3}{9 - 3} = \frac{12 + 6\sqrt{3}}{6} \\ &= 2 + \sqrt{3} \end{aligned}$$

3) $\cos(195^\circ) = \cos(150^\circ + 45^\circ)$

$$\begin{aligned} &= \cos 150^\circ \cos 45^\circ - \sin 150^\circ \sin 45^\circ \\ &= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{-\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

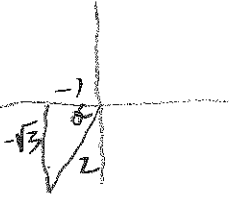
4) $\sin(165^\circ)$

$$\begin{aligned} &= \sin(120^\circ + 45^\circ) \\ &= \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$



Section 5.4 – Review of Sum and Difference Formulas

$$\begin{aligned}
 5) \sin(-75^\circ) &= \sin(285^\circ) \\
 &= \sin(240^\circ + 45^\circ) \\
 &= \sin 240^\circ \cos 45^\circ + \cos 240^\circ \sin 45^\circ \\
 &= \frac{-\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{-1}{2} \cdot \frac{\sqrt{2}}{2} \\
 &= \frac{-\sqrt{6} - \sqrt{2}}{4}
 \end{aligned}$$



$$\begin{aligned}
 6) \tan(345^\circ) &= \tan(315^\circ + 30^\circ) \\
 &= \frac{\tan 315^\circ + \tan 30^\circ}{1 - \tan 315^\circ \tan 30^\circ} \\
 &= \frac{-1 + \frac{\sqrt{3}}{3}}{1 - (-1)(\frac{\sqrt{3}}{3})} = \frac{\frac{-3 + \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}} = \frac{\sqrt{3} - 3}{\sqrt{3} + 3} \cdot \frac{\sqrt{3} - 3}{\sqrt{3} - 3} \\
 &= \frac{3 - 6\sqrt{3} + 9}{3 - 9} = \frac{12 - 6\sqrt{3}}{-6} \\
 &= -2 + \sqrt{3}
 \end{aligned}$$



$$\begin{aligned}
 7) \cos(285^\circ) &= \cos(240^\circ + 45^\circ) \\
 &= \cos 240^\circ \cos 45^\circ - \sin 240^\circ \sin 45^\circ \\
 &= \frac{-1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{-\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\
 &= \frac{-\sqrt{2} + \sqrt{6}}{4}
 \end{aligned}$$

$$\begin{aligned}
 8) \sin(105^\circ) &= \sin(45^\circ + 60^\circ) \\
 &= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ \\
 &= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \\
 &= \frac{\sqrt{2} + \sqrt{6}}{4}
 \end{aligned}$$

Write as sine, cosine, or tangent of an angle. You do NOT have to find the value:

$$\begin{aligned}
 9) \sin(42^\circ)\cos(17^\circ) - \cos(42^\circ)\sin(17^\circ) \\
 &= \sin(42^\circ - 17^\circ) \\
 &= \sin 25^\circ
 \end{aligned}$$

$$\begin{aligned}
 10) \cos 45^\circ \cos 120^\circ - \sin 45^\circ \sin 120^\circ \\
 &= \cos(45^\circ + 120^\circ) = \cos 165^\circ
 \end{aligned}$$

Section 5.4 – Review of Sum and Difference Formulas

$$11) \frac{\tan(19^\circ) + \tan(47^\circ)}{1 - \tan(19^\circ)\tan(47^\circ)}$$

$$\begin{aligned} & \tan(19+47) \\ & = \tan(66^\circ) \end{aligned}$$

$$12) \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$$

$$= \sin(60-45) = \sin 15^\circ$$

$$13) \cos(94^\circ)\cos(18^\circ) + \sin(94^\circ)\sin(18^\circ)$$

$$\begin{aligned} & = \cos(94-18) \\ & = \cos 76^\circ \end{aligned}$$

$$14) \frac{\tan 25^\circ + \tan 10^\circ}{1 - \tan 25^\circ \tan 10^\circ}$$

$$\begin{aligned} & = \tan(25+10) \\ & = \tan 35^\circ \end{aligned}$$

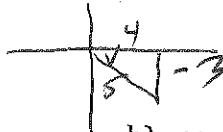
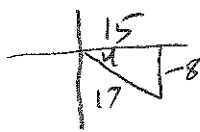
$$15) \frac{\tan 68^\circ - \tan 115^\circ}{1 + \tan 68^\circ \tan 115^\circ}$$

$$\begin{aligned} & = \tan(68-115) \\ & = \tan(-47^\circ) \end{aligned}$$

Section 5.4 – Review of Sum and Difference Formulas

16) Find the EXACT value of the trig function given that

$$\sin u = \frac{-8}{17}, \quad \cos v = \frac{4}{5} \quad \text{and both angles are in Quadrant IV}$$



a) $\tan(u-v)$

$$= \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$= \frac{\frac{-8}{17} - \frac{-3}{4}}{1 + \left(\frac{-8}{17}\right)\left(\frac{-3}{4}\right)} = \frac{\frac{-32 + 45}{60}}{\frac{60 + 24}{60}}$$

$$= \frac{13}{84}$$

b) $\cos(u+v)$

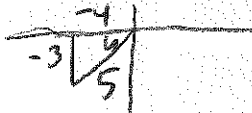
$$= \cos u \cos v - \sin u \sin v$$

$$= \frac{15}{17} \cdot \frac{4}{5} - \left(\frac{-8}{17}\right)\left(\frac{-3}{5}\right)$$

$$= \frac{60}{85} - \frac{24}{85} = \frac{36}{85}$$

17) Find the exact value of the trig function given that

$$\sin u = \frac{-3}{5} \quad \text{and} \quad \cos v = \frac{-5}{13} \quad \text{where both } u \text{ and } v \text{ are in Quadrant III.}$$



a) $\sin(u-v)$

$$= \sin u \cos v - \cos u \sin v$$

$$= \frac{-3}{5} \cdot \frac{-5}{13} - \frac{-4}{5} \cdot \frac{-12}{13}$$

$$= \frac{15}{65} - \frac{48}{65} = \frac{-33}{65}$$

b) $\cos(u-v)$

$$= \cos u \cos v + \sin u \sin v$$

$$= \frac{-4}{5} \cdot \frac{-5}{13} + \frac{-3}{5} \cdot \frac{-12}{13}$$

$$= \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$$

c) $\tan(u+v)$

$$= \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$= \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4} \cdot \frac{12}{5}}$$

$$= \frac{15 + 48}{20}$$

$$= \frac{20 - 36}{20}$$

$$= \frac{63}{-16}$$