Sections 5.4 & 5.5 Additional Practice (for homework!)

Here are all the identities I will give you on the test:

Reciprocal Identities

$\sin u = \frac{1}{\csc u}$ $\cos u = \frac{1}{\sec u}$ $\tan u = \frac{1}{\cot u}$

$\csc u = \frac{1}{\sin u}$ $\sec u = \frac{1}{\cos u}$ $\cot u = \frac{1}{\tan u}$

Quotient Identities

$\tan u = \frac{\sin u}{\cos u}$ $\cot u = \frac{\cos u}{\sin u}$

Co-function Identities

$\sin\left(\frac{\pi}{2} - u\right) = \cos u$

$\cos\left(\frac{\pi}{2} - u\right) = \sin u$

$\tan\left(\frac{\pi}{2} - u\right) = \cot u$

$\cot\left(\frac{\pi}{2} - u\right) = \tan u$

$\sec\left(\frac{\pi}{2} - u\right) = \csc u$

$\csc\left(\frac{\pi}{2} - u\right) = \sec u$

Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$ $1 + \tan^2 u = \sec^2 u$ $1 + \cot^2 u = \csc^2 u$

Odd & Even Identities

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$

$\csc(-x) = -\csc x$ $\sec(-x) = \sec x$ $\cot(-x) = -\cot x$

Sum & Difference Formulas

$$\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$$

$$\sin(\theta - \beta) = \sin\theta\cos\beta - \cos\theta\sin\beta$$

$$\cos(\theta + \beta) = \cos\theta\cos\beta - \sin\theta\sin\beta$$

$$\cos(\theta - \beta) = \cos\theta\cos\beta + \sin\theta\sin\beta$$

$$\tan(\theta + \beta) = \frac{\tan\theta + \tan\beta}{1 - \tan\theta\tan\beta}$$

$$\tan(\theta - \beta) = \frac{\tan\theta - \tan\beta}{1 + \tan\theta\tan\beta}$$

Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1 - \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 + \cos\theta}$$

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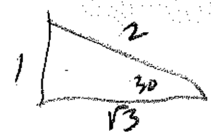
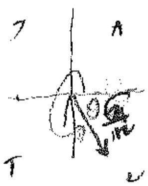


Using any of the above formulas, Find the <u>exact</u> value of the following (show your work):

1. $\sin\frac{19\pi}{12} = -\sin\frac{5\pi}{12}$



$= -\sin(75^\circ) = -\sin(45+30)$

$= -\sin 45\cos 30 + \cos 45 \sin 30$

$= -\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$

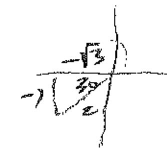
$= \frac{-\sqrt{6}-\sqrt{2}}{4}$

2. $\cos 255^\circ = \cos(210+45)$

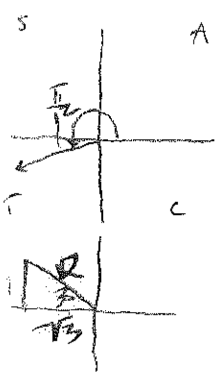
$= \cos 210\cos 45 - \sin 210 \sin 45$

$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - -\frac{1}{2}\frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{6}+\sqrt{2}}{4}$



3. $\tan\frac{13\pi}{12} = \tan(195) = \tan(150+45)$



$= \frac{\tan 150 + \tan 45}{1 - \tan 150 \tan 45}$

$= \frac{-\frac{\sqrt{3}}{3}+1}{1-\left(-\frac{\sqrt{3}}{3}\cdot 1\right)} = \frac{\frac{-\sqrt{3}+3}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$

$= \frac{9-6\sqrt{3}+3}{9-3} = \frac{12-6\sqrt{3}}{6} = 2-\sqrt{3}$

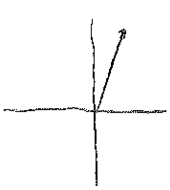
4. $\tan 67.5^\circ = \tan\frac{135}{2}$

$= \frac{1-\cos 135}{\sin 135}$

$= \frac{1-\frac{-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2+\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{2\sqrt{2}+2}{2} = \sqrt{2}+1$

5. $\cos\frac{5\pi}{12} = \cos 75^\circ = \cos(30+45)$



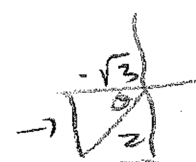
$= \cos 30\cos 45 - \sin 30 \sin 45$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

6. $\tan 105^\circ = \tan\frac{210}{2}$

$= \frac{1-\cos 210}{\sin 210}$



$= \frac{1-\frac{-\sqrt{3}}{2}}{-1/2} = \frac{2+\sqrt{3}}{-1} = -2-\sqrt{3}$

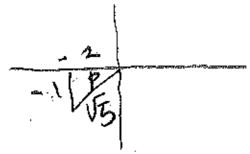
7. There is no formula given for the cotangent of a sum or difference. How could you find the exact value of $\cot 15^\circ$?

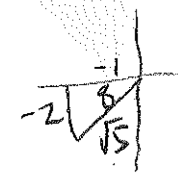
$\frac{1}{\tan 15^\circ}$ find reciprocal of $\tan 15^\circ = \tan(45-30) = \frac{\tan 45 - \tan 30}{1+\tan 45\tan 30}$

$= \frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right) = \frac{9-6\sqrt{3}+3}{9-3} = \frac{12-6\sqrt{3}}{6} = 2-\sqrt{3}$

$\therefore \cot 15^\circ = \frac{1}{2-\sqrt{3}}\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right) = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

Sections 5.4 & 5.5 Additional Practice (for homework!)





8. Find the values of the following, given $\tan p = \frac{1}{2}$, $\sec q = -\sqrt{5}$; p & q in QIII

$\cos = -\frac{1}{\sqrt{5}}$

$\sin 2q =$ $2\sin q \cos q$

$= 2\left(\frac{-2}{\sqrt{5}}\right)\left(\frac{-1}{\sqrt{5}}\right)$

$= +\frac{4}{5}$

$\tan\frac{p}{2} =$ $\frac{1-\cos p}{\sin p} = \frac{1 - \frac{-2}{\sqrt{5}}}{\frac{-1}{\sqrt{5}}}$

$= \frac{\sqrt{5}+2}{\sqrt{5}} \cdot \frac{-\sqrt{5}}{1} = -\sqrt{5}-2$

$\cos(p+q) =$ $\cos p \cos q - \sin p \sin q$

$= \frac{-2}{\sqrt{5}} \cdot \frac{-1}{\sqrt{5}} - \frac{+1}{\sqrt{5}} \cdot \frac{-2}{\sqrt{5}}$

$= \frac{2-2}{5} = 0$

$\sin(p-q) =$ $\sin p \cos q - \cos p \sin q$

$= \frac{-1}{\sqrt{5}} \cdot \frac{-1}{\sqrt{5}} - \frac{-2}{\sqrt{5}} \cdot \frac{-2}{\sqrt{5}}$

$= \frac{1}{5} - \frac{4}{5} = -\frac{3}{5}$

9. Which expressions are equal to $\sin 15°$? (There may be more than one correct choice)

A. $\sin 45°\cos 30° + \cos 45°\sin 30°$ $= \sin 75$

B. $\sin 45°\cos 30° - \cos 45°\sin 30°$

C. $\sin 60°\cos 45° - \cos 60°\sin 45°$

D. $\cos 60°\cos 45° - \sin 60°\sin 45°$ $= \cos 105°$

Sections 5.4 & 5.5 Additional Practice (for homework!)

10. Simplify the following to one trig expression using any of the formulas:

$\sin 3x\cos 2x+\cos 3x\sin 2x$ $= \sin(3x+2x) = \sin 5x$

$\cos 37°\cos 22°-\sin 37°\sin 22°$ $= \cos(37+22) = \cos 59°$

$\sin 10°\cos 5°+\cos 10°\sin 5°$ $= \sin 15°$

$\dfrac{\tan 5x-\tan 4x}{1+\tan 5x\tan 4x}$ $= \tan(5x-4x) = \tan x$

$\cos 5x\cos x-\sin 5x\sin x$ $= \cos 6x$

$\cos 15°\cos 75°-\sin 15°\sin 75°$ $= \cos 90°$

$\cos 4x\cos 3x+\sin 4x\sin 3x$ $= \cos x$

$\sin 2x\cos x-\sin x\cos 2x$ $= \sin x$

Sections 5.4 & 5.5 Additional Practice (for homework!)

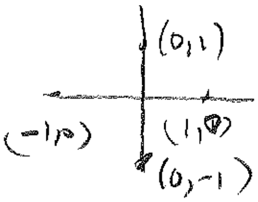
Find the solutions in the interval $[0,2\pi)$ (think back to section 5.3)

11. $\sin 3x\cos 2x+\cos 3x\sin 2x=1$

$\sin 5x = 1$

$5x = \frac{\pi}{2}$

$x = \frac{\pi}{10}$



12. $\sin 2x=\sin x$

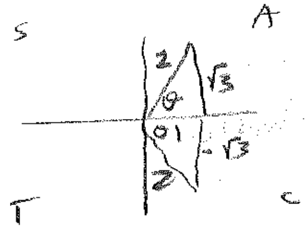
$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \qquad 2(\cos x - 1) = 0$

$\cos x = \frac{1}{2}$

$x = 0, \pi \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$



13. $\sin 2x-\cos x=0$

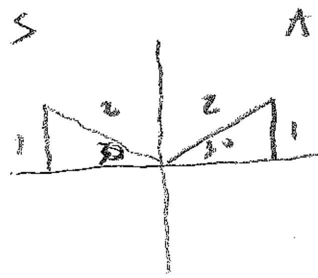
$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \qquad 2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$



14. $\cos 2x+2=-4\cos x-2\cos^2 x$

$(2\cos^2 x - 1) + 2 + 4\cos x + 2\cos^2 x = 0$

$4\cos^2 x + 4\cos x + 1 = 0$

$(2\cos x + 1)(2\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$