Sections 5.4 & 5.5 – I.C.E – Solving Trig Equations

Here are all the identities I will give you on the test:

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$
$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Co-function Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$
$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$
$$\tan\left(\frac{\pi}{2} - u\right) = \cot u$$
$$\cot\left(\frac{\pi}{2} - u\right) = \tan u$$
$$\sec\left(\frac{\pi}{2} - u\right) = \csc u$$
$$\csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \quad 1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$

Odd & Even Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$
$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum & Difference Formulas

$$\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$$

$$\sin(\theta - \beta) = \sin\theta\cos\beta - \cos\theta\sin\beta$$

$$\cos(\theta + \beta) = \cos\theta\cos\beta - \sin\theta\sin\beta$$

$$\cos(\theta - \beta) = \cos\theta\cos\beta + \sin\theta\sin\beta$$

$$\tan(\theta + \beta) = \frac{\tan\theta + \tan\beta}{1 - \tan\theta\tan\beta}$$

$$\tan(\theta - \beta) = \frac{\tan\theta - \tan\beta}{1 + \tan\theta\tan\beta}$$

Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1 - \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 + \cos\theta}$$

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Using any of the above formulas, Find the <u>exact</u> value of the following (show your work):

1. $\cos 15°$

2. $\sin \frac{13\pi}{12}$

3. $\tan \frac{17\pi}{12}$

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4. Write the following as a **single trigonometric** function:

$2\sin A\cos A =$

$\dfrac{2\tan B}{1-\tan^2 B} =$

$1-2\sin^2 C =$

$2\sin 105^\circ \cos 105^\circ =$

$\cos^2 15^\circ - \sin^2 15^\circ =$

$\dfrac{2\tan 67.5^\circ}{1-\tan^2 67.5^\circ} =$

$2\cos^2 112.5^\circ - 1 =$

$\sin 75^\circ \cos 15^\circ + \sin 15^\circ \cos 75^\circ =$

$\dfrac{\tan 52.5^\circ + \tan 7.5^\circ}{1-\tan 52.5^\circ \tan 7.5^\circ} =$

$\cos 127.5^\circ \cos 7.5 + \sin 127.5^\circ \sin 7.5^\circ =$

$\sin\left(\dfrac{\pi}{12}\right)\cos\left(\dfrac{\pi}{12}\right) =$

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5. Find the exact values of the following. It would be helpful to draw a picture!

$$\sin x = \frac{5}{13},\ \cos y = \frac{-8}{17};\ x \text{ \& } y \text{ in QII}$$

$\cos 2x =$ $\tan 2x =$

$\sin(x+y) =$ $\tan(y-x) =$

6. Matching (Some of the expressions on the right may be used more than once or not at all).

a. $\sin(\alpha-\beta)$	1. $\sin\beta$
b. $\cos(\alpha+\beta)$	2. $\sin\alpha\cos\beta+\cos\alpha\sin\beta$
c. $\sin(180°+\beta)$	3. $-\cos\beta$
d. $\sin(180°-\beta)$	4. $\cos\alpha\cos\beta+\sin\alpha\sin\beta$
e. $\cos(180°+\beta)$	5. $\sin\alpha\cos\beta-\cos\alpha\sin\beta$
f. $\sin(\alpha+\beta)$	6. $\cos\alpha\cos\beta-\sin\alpha\sin\beta$
g. $\cos(90°-\beta)$	7. $-\sin\beta$
h. $\cos(\alpha-\beta)$	8. $\cos\beta$

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Find the **solutions** in the interval $[0, 2\pi)$ (think back to section 5.3)

7. $\sin 2x\cos x + \cos 2x\sin x = -\frac{1}{2}$

8. $\cos 2x\cos x - \sin 2x\sin x = -\frac{\sqrt{3}}{2}$

9. $\sin 2x + 2\cos x = 0$