

Pre-Calculus CP 1 – Section 9.5 ICE
 Pascal's Triangle & The Binomial Theorem

Name: KEY

Use Pascal's Triangle and/or The Binomial Theorem to expand and find information about the following. Show all work, including work you may enter in your calculator.

- 1) Use Pascal's Triangle (or your calculator) to find the 5th term in the 8th row.

$${}^8 C_4 = \boxed{70}$$

- 2) Use Pascal's Triangle (or your calculator) to find the 13th term in the 20th row.

$${}^{20} C_{12} = \boxed{125,970}$$

- 3) Expand $(3x-4)^5$ using Pascal's Triangle and/or the Binomial Theorem.

$$\begin{aligned}
 &= {}^5 C_0 (3x)^5 + {}^5 C_1 (3x)^4 (-4) + {}^5 C_2 (3x)^3 (-4)^2 + {}^5 C_3 (3x)^2 (-4)^3 + {}^5 C_4 (3x)^1 (-4)^4 + {}^5 C_5 (-4)^5 \\
 &= \boxed{243x^5 - 1620x^4 + 4320x^3 - 5760x^2 + 3840x - 1024}
 \end{aligned}$$

- 4) Expand $(2x+5y)^7$ using Pascal's Triangle and/or the Binomial Theorem.

$$\begin{aligned}
 &1(2x)^7 (5y)^0 + 7(2x)^6 (5y)^1 + 21(2x)^5 (5y)^2 + 35(2x)^4 (5y)^3 + 35(2x)^3 (5y)^4 \\
 &+ 21(2x)^2 (5y)^5 + 7(2x)^1 (5y)^6 + 1(2x)^0 (5y)^7
 \end{aligned}$$

$$\begin{aligned}
 &= \boxed{128x^7 + 2240x^6y + 16800x^5y^2 + 70000x^4y^3 + 175000x^3y^4 + 262500x^2y^5} \\
 &\quad + 218750xy^6 + 78125y^7
 \end{aligned}$$

- 5) Find the term containing x^3 in the expansion of $(7x+3)^9$ using Pascal's Triangle and/or the Binomial Theorem. \leftarrow 10 terms

$$\begin{aligned}
 & x^3 y^6 \leftarrow 7^{\text{th}} \text{ term} \\
 & {}_9 C_6 (7x)^3 (3)^6 = 84 (343x^3) (729) \\
 & = 21,003,948 x^3
 \end{aligned}$$

- 6) Find the coefficient a of the term ax^2y^8 in the expansion of $(4x-y)^{10}$. \leftarrow 9th term of \leftarrow 11 terms

$$\begin{aligned}
 & {}_{10} C_8 (4x)^2 (-y)^8 = 45 (16x^2) (y^8) \\
 & = 720x^2y^8
 \end{aligned}$$

- 7) Find the **coefficient** of the fifth term in the expansion of $(-3x+y)^{14}$ using Pascal's Triangle and/or the Binomial Theorem.

$$\begin{aligned}
 & {}_{14} C_4 (-3x)^{10} (y)^4 = 1001 (59,049 x^{10}) (y^4) \\
 & = 59,108,049 x^{10} y^4
 \end{aligned}$$

8) Express $1296x^{12} - 4320x^9y^2 + 5400x^6y^4 - 3000x^3y^6 + 625y^8$ in the form $(a+b)^n$.

5 terms \Rightarrow 4th row
 alternating signs $\Rightarrow (a-b)^n$

$$1296x^{12} = a^4$$

$$\Rightarrow a = \sqrt[4]{1296x^{12}} = 6x^3$$

$$625y^8 = b^4$$

$$\Rightarrow b = \sqrt[4]{625y^8} = 5y^2$$

$(6x^3 - 5y^2)^4$

9) Express

$$117,649x^{18} + 1,008,420x^{15}y^4 + 3,601,500x^{12}y^8 + 6,860,000x^9y^{12} + 7,350,000x^6y^{16} + 4,200,000x^3y^{20} + 1,000,000y^{24}$$

in the form $(a+b)^n$.

7 terms \Rightarrow 6th row
 all positive signs $\Rightarrow (a+b)^n$

$$a^6 = 117,649x^{18}$$

$$\Rightarrow a = \sqrt[6]{117,649x^{18}} = 7x^3$$

$$b^6 = 1,000,000y^{24}$$

$$\Rightarrow b = \sqrt[6]{1,000,000y^{24}} = 10y^4$$

$(7x^3 + 10y^4)^6$