Pre-Calculus CP 1 I.C.E **Name:** KEY

Limits – Visual, Algebraic, to Infinity (…and beyond!!)

1)Find the following limits given the graph of $h(x)$

a) $\lim\limits_{x\to 12^-} h(x) =$ 3

b) $\lim\limits_{x\to 12^+} h(x) =$ 3

c) $\lim\limits_{x\to 12} h(x) =$ 3

d) $\lim\limits_{x\to 3^-} h(x) =$ ∞

10

e) $\lim\limits_{x\to 3^+} h(x) =$ ∞

f) $\lim\limits_{x\to 3} h(x) =$ ∞

g) $\lim\limits_{x\to 2^-} h(x) =$ 1

h) $\lim\limits_{x\to 2^+} h(x) =$ 1

i) $\lim\limits_{x\to 2} h(x) =$ 1

j) $\lim\limits_{x\to 4^-} h(x) =$ 6

-4

k) $\lim\limits_{x\to 4^+} h(x) =$ 6

l) $\lim\limits_{x\to 4} h(x) =$ 6

m) $\lim\limits_{x\to -2^-} h(x) =$ 2

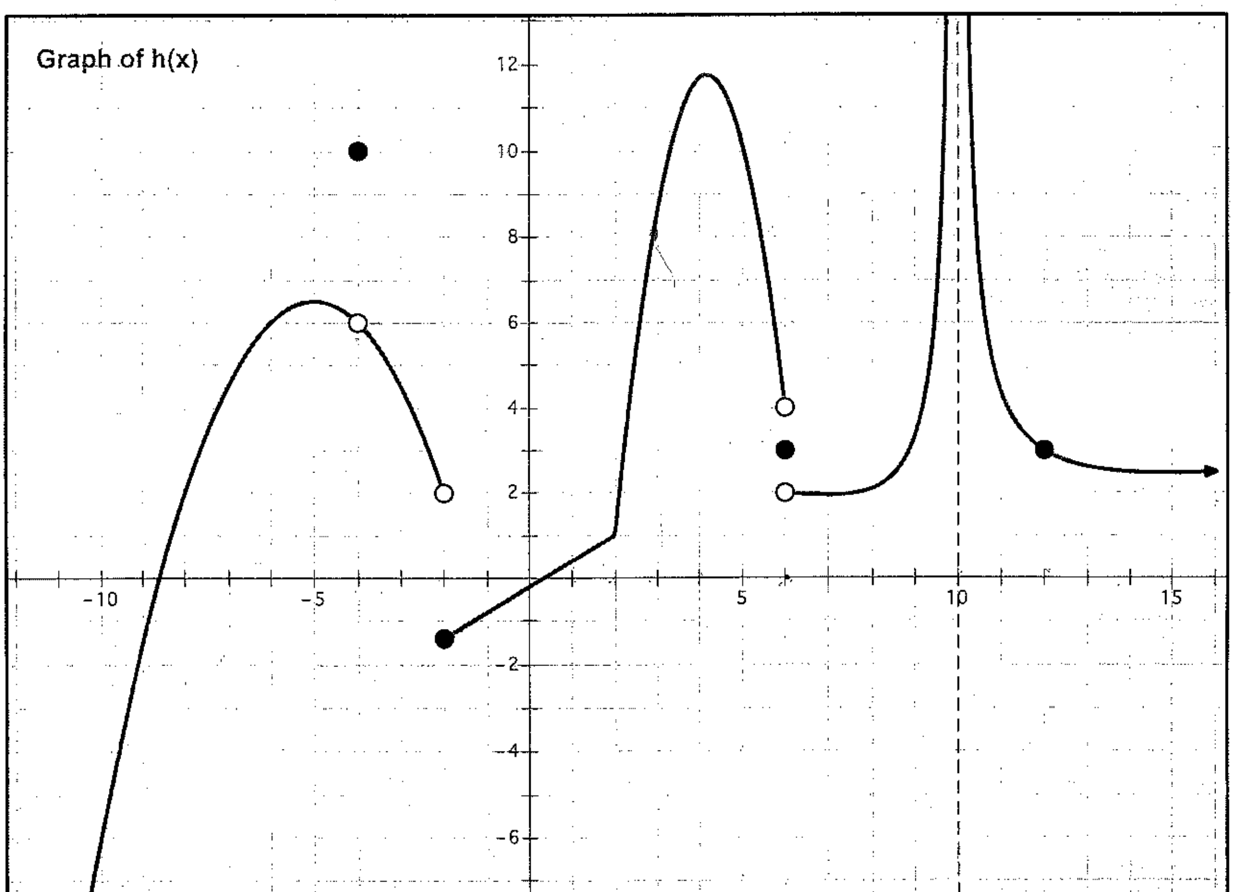
n) $\lim\limits_{x\to -2^+} h(x) =$ -1.5

o) $\lim\limits_{x\to -2} h(x) =$ DNE

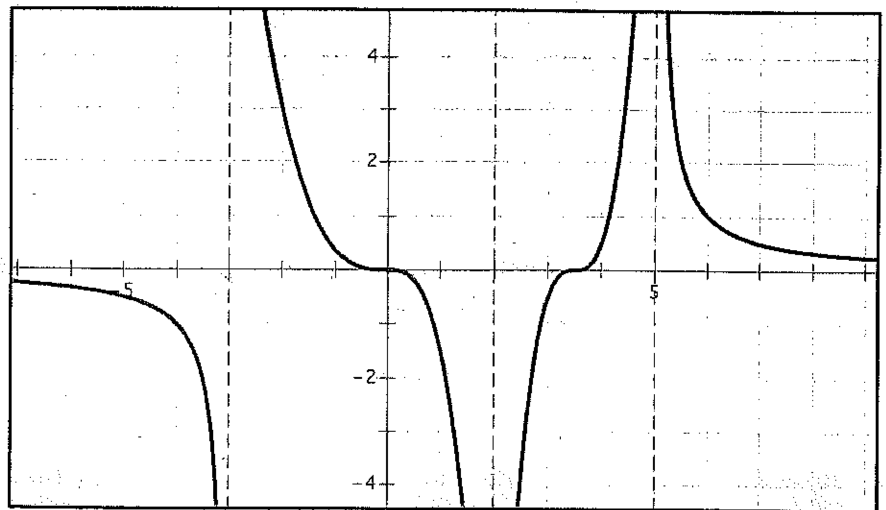
p) $\lim\limits_{x\to 6^-} h(x) =$ 4

q) $\lim\limits_{x\to 6^+} h(x) =$ 2

r) $\lim\limits_{x\to 6} h(x) =$ DNE



2) Find the limits below based on the graph:



a) $\lim\limits_{x \to 2} R(x) = -\infty$

b) $\lim\limits_{x \to 5} R(x) = \infty$

c) $\lim\limits_{x \to \infty} R(x) = 0$

d) $\lim\limits_{x \to 3^-} R(x) = -\infty$

-3

e) $\lim\limits_{x \to 3^+} R(x) = \infty$

f) The equations for the vertical asymptotes are: $x=-3$, $x=2$ and $x=5$

3) Find the following infinite limits:

$$\lim_{x \to \infty} \frac{-x+3}{x^2+3} = 0$$

$$\lim_{x \to \infty} \frac{5x^6 - x^5}{x^3+1} = \infty$$

$$\lim_{x \to \infty} \frac{3x^2}{2x+1} = \infty$$

$\lim\limits_{x\to\infty}\frac{4x^2+1}{-3x^2+5}=$ $-\frac{4}{3}$

4) Find the following limits algebraically. Watch out for holes and vertical asymptotes, as answering those will require more work. Be sure to show your analysis and be careful with notation!

a) $\lim\limits_{x\to-2}\frac{(x-4)^2}{x+2}=$ $\frac{(-6)^2}{0}$

$\lim\limits_{x\to-2^-}\frac{(x-4)(x-4)}{x+2}=\frac{+}{-}=-\infty$

$\lim\limits_{x\to-2^+}\frac{(x-4)^2}{x+2}=\frac{+}{+}=+\infty$

$\lim\limits_{x\to-2}$ $\boxed{DNE}$

b) $\lim_{x\to -4}\frac{\sqrt{x+20}-4}{x+4}=$ $\frac{0}{0}$ hole! $\left(\frac{\sqrt{x+20}+4}{\sqrt{x+20}+4}\right)=\frac{x+20-16}{(x+4)(\sqrt{x+20}+4)}=\frac{\cancel{x+4}}{\cancel{(x+4)}(\sqrt{x+20}+4)}$

$=\lim_{x\to -4}\frac{1}{\sqrt{x+20}+4}=\frac{1}{\sqrt{16}+4}=\frac{1}{8}$

c) $\lim_{x\to 4}\frac{4+x}{x^2-16}=$ $\frac{8}{0}$

$\lim_{x\to 4^-}\frac{4+x}{x^2-16}=\frac{+}{-}=-\infty$

$\lim_{x\to 4^+}\frac{4+x}{x^2-16}=\frac{+}{+}=+\infty$

$\lim_{x\to 4}\frac{4+x}{x^2-16}=\boxed{DNE}$

d) $\lim_{x\to 3}\frac{\frac{1}{x+3}-\frac{1}{6}}{x-3}=$ $\frac{0}{0}$ hole! $\frac{\frac{6}{(x+3)6}-\frac{x+3}{(x+3)6}}{x-3}=\frac{\frac{6-(x+3)}{6(x+3)}}{x-3}=\frac{\overset{-1}{\cancel{3-x}}}{6(x+3)}\cdot\frac{1}{\cancel{x-3}}$

$=\lim_{x\to 3}\frac{-1}{6(x+3)}=-\frac{1}{36}$

e) $\lim_{x\to 7}\frac{x^2+5x-14}{49-x^2}=$ $\frac{\pm}{0}$

$\lim_{x\to 7^-}\frac{x^2+5x-14}{49-x^2}=\frac{+}{+}=+\infty$

$\lim_{x\to 7^+}\frac{x^2+5x-14}{49-x^2}=\frac{+}{-}=-\infty$

$\lim_{x\to 7}=\boxed{DNE}$

f) $\lim_{x\to 7}(-5)=$ -5

g) $\lim\limits_{x \to \pi/2}(2\sin x + 2) =$ $2\sin\frac{\pi}{2} + 2 = 4$

h) $\lim\limits_{x \to 3^-} \dfrac{-4x^2 - 2}{x - 3} =$ $\dfrac{-}{-} = +\infty$

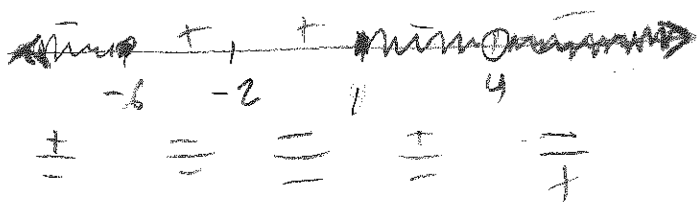
i) $\lim\limits_{x \to 5} \dfrac{-x+1}{(x-5)^3} =$ $\dfrac{-5+1}{0}$

$\left.\begin{array}{l} \lim\limits_{x \to 5^-} \dfrac{-x+1}{(x-5)^3} = \dfrac{-}{-} = +\infty \\ \lim\limits_{x \to 5^+} \dfrac{-x+1}{(x-5)^3} = \dfrac{-}{+} = -\infty \end{array}\right\} \boxed{\text{DNE}}$

j) $\lim\limits_{x \to 4} \dfrac{x^2 - 3}{x + 4} =$ $\dfrac{4^2 - 3}{4+4} = \dfrac{13}{8}$

5)

a) Use a number line to solve the inequality: $\frac{-3(x-4)(x-1)(x+6)}{(x-4)(x+2)^2} \le 0$



b) Sketch the graph of $f(x)=\frac{-3(x-4)(x-1)(x+6)}{(x-4)(x+2)^2}$ $\frac{-3(-4)(-1)(6)}{(-4)(2)^2} = \frac{18}{4} = \frac{9}{2}$

Hole? Yes   If yes, where? $x=4$

HA / SA? $y=-3$

VA? $x=-2$

x-int? $(1,0)$, $(-6,0)$

y-int? $(0, 9/2)$