Inverse Functions

Ex 1) Let **f(x)** be defined by the set of ordered pairs:
{(-1, -5), (0, 3), (2, 1) and (3, 3)}

Then, **$f^{-1}(x)$** "f inverse" would be defined as: {(-5, -1), (3, 0), (1, 2) and (3, 3)}

a. What is the **domain** of f(x)? {-1, 0, 2, 3}

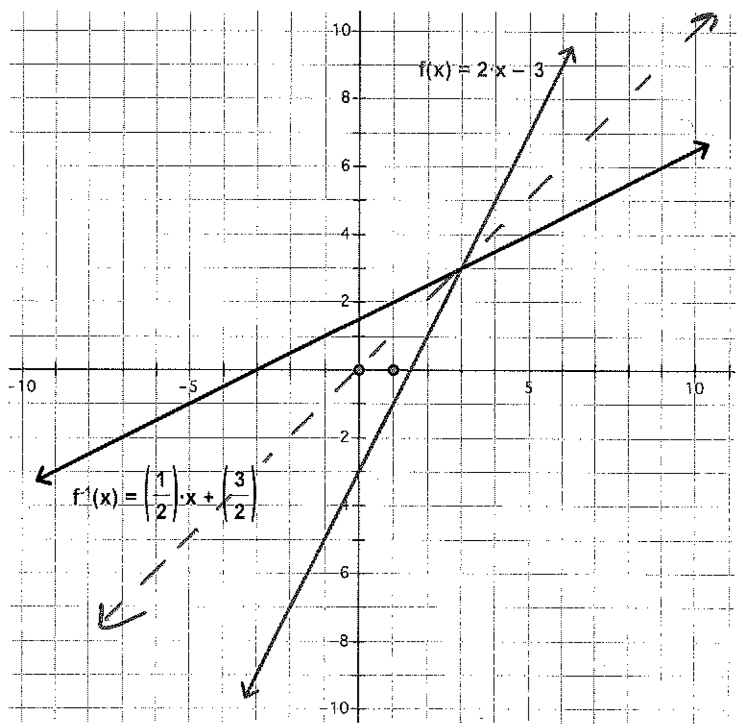
b. What is the **range** of $f^{-1}(x)$? {-1, 0, 2, 3}

Therefore, the **domain** of f(x) is always equal to the **range** of $f^{-1}(x)$ and vice versa!

c. Let's graph f(x) and $f^{-1}(x)$ as defined below:

$f(x) = 2x - 3$

$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$



d. Draw in the line to which these graphs are symmetric.

e. What is the equation of this line? y = x

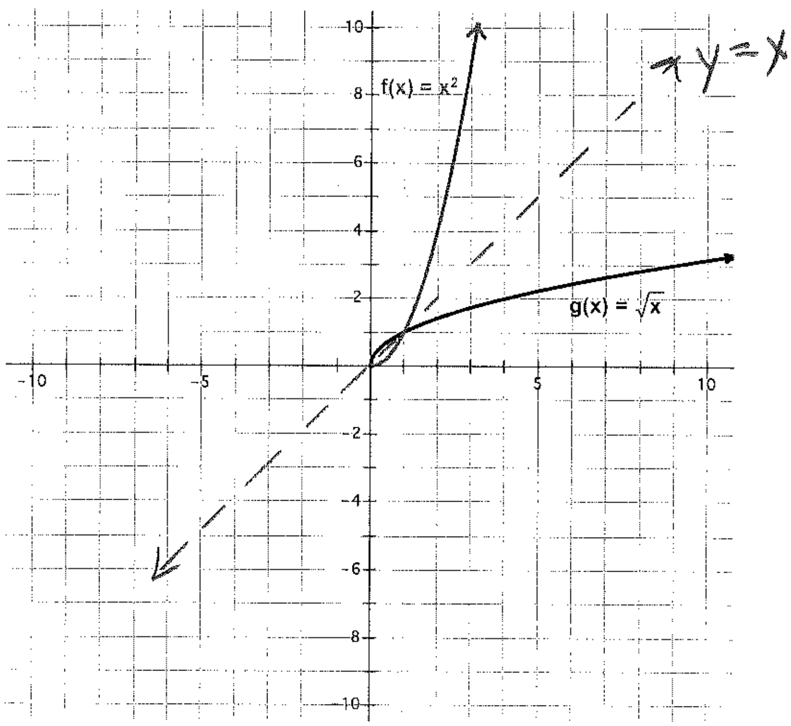
Ex 2) Sketch the graphs of the inverse functions $f(x) = x^2$ $(x \geq 0)$ and $f^{-1}(x) = \sqrt{x}$ on the same set of axes and show that the graphs are reflections of each other in the line $y = x$.

$f(x) = x^2$

x	0	1	2	3
y	0	1	4	9

$f^{-1}(x) = \sqrt{x}$

x	0	1	4	9
y	0	1	3	3



Finding an Inverse Function Algebraically

To find an inverse function algebraically, we switch **x** and **y** and solve for **y**!

Ex. 3) Find the inverse function of f(x) = $\frac{x+3}{x-2}$, algebraically.

$$y=\frac{x+3}{x-2} \rightarrow x=\frac{y+3}{y-2}$$

$$\Rightarrow x(y-2)=y+3$$
$$\Rightarrow xy-2x=y+3$$
$$\Rightarrow xy-y=2x+3$$
$$\Rightarrow y(x-1)=2x+3$$
$$\Rightarrow y=\frac{2x+3}{x-1}$$

Try this!

Ex 4) Find the inverse function of f(x) = $\frac{5-3x}{2}$, algebraically.

$$x=\frac{5-3y}{2}$$
$$2x=5-3y$$
$$3y=5-2x$$
$$y=\frac{5-2x}{3}$$
$$\boxed{f^{-1}(x)=\frac{5-2x}{3}}$$

Verifying Inverse Functions Algebraically

Let f and g be two functions such that f(g(x)) = x for every x in the domain of g, and g(f(x)) = x for every x in the domain of f.

Then g(x) is the **inverse function** of f(x) and g(x) can be denoted by $\mathbf{f^{-1}(x)}$.

Examples

Verify that f(x) and g(x) are inverse functions algebraically

a. $f(x)=3-4x$ $\qquad g(x)=\dfrac{3-x}{4}$

1st: Find f(g(x)):

$$\begin{aligned} f(g(x)) &= 3-4\left(\frac{3-x}{4}\right) \\ &= 3-1(3-x) \\ &= 3-3+x \\ &= x \end{aligned}$$

2nd: Find g(f(x)): **Try this!!**

$$g(f(x)) = \frac{3-(3-4x)}{4}$$

$$= \frac{3-3+4x}{4} = \frac{4x}{4} = x$$

b. Verify that f(x) and g(x) are inverse functions algebraically (in one direction only)

$$f(x) = \frac{x+1}{x-2} \qquad g(x) = \frac{2x+1}{x-1}$$

$$\begin{aligned} f(g(x)) &= \frac{\left(\frac{2x+1}{x-1}\right)+1}{\left(\frac{2x+1}{x-1}\right)-2} \\ &= \frac{(2x+1)+1(x-1)}{(2x+1)-2(x-1)} \\ &= \frac{3x}{2x+1-2x+2} \\ &= \frac{3x}{3} \\ &= x \end{aligned}$$

One-to-One Functions

A function is "one-to-one" if for every **y** there is exactly one **x.**

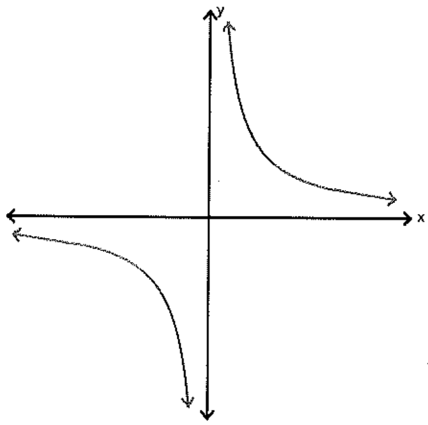
A quick way to tell is by applying the **horizontal** line test.

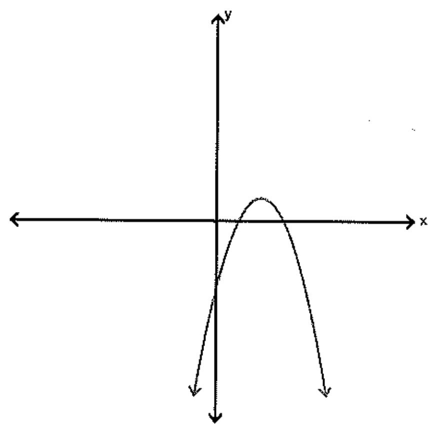
Horizontal Line Test

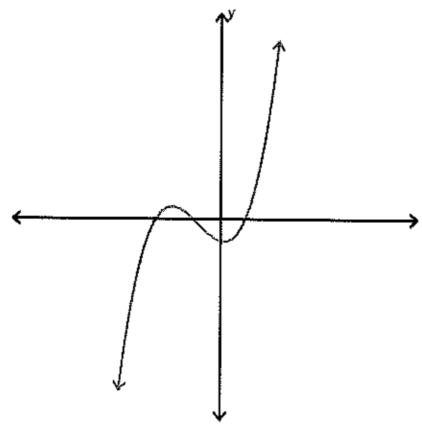
A function f has an inverse function iff no **horizontal** line intersects the graph of f at **more than one** point.

→ A function f has an inverse function iff f is **one-to-one.**

Examples







a.

One to one? Yes

Has inverse? Yes

b.

One to one? NO

Has inverse? NO

c.

One to one? NO

Has inverse? NO

Suggested Practice Problems: p.99-101 (these are all odds and answers in back of the book) #1, 3, 13-23 (part a), 25, 29, 31, 39, 43, 45, 47, 51, 55, 57, 65, 67