

Inverse Functions

Ex 1) Let $f(x)$ be defined by the set of ordered pairs:
 $\{(-1, -5), (0, 3), (2, 1) \text{ and } (3, 3)\}$

Then, $f^{-1}(x)$ “f inverse” would be defined as: $\{(-5, -1), (3, 0), (1, 2) \text{ and } (3, 3)\}$

a. What is the **domain** of $f(x)$? $\{-1, 0, 2, 3\}$

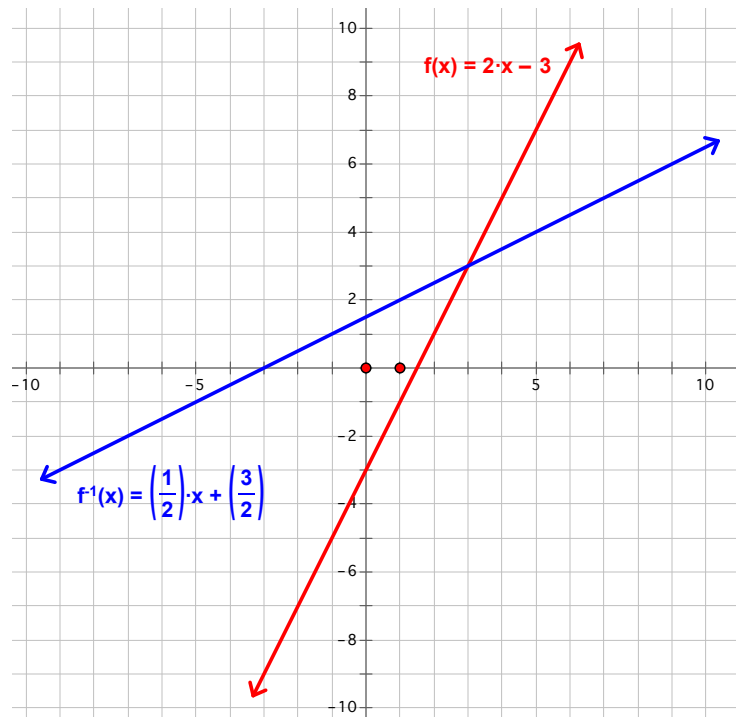
b. What is the **range** of $f^{-1}(x)$? $\{-1, 0, 2, 3\}$

Therefore, the **domain** of $f(x)$ is always equal to the **range** of $f^{-1}(x)$ and vice versa!

c. Let's graph $f(x)$ and $f^{-1}(x)$ as defined below:

$$f(x) = 2x - 3$$

$$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$



d. Draw in the line to which these graphs are symmetric.

e. What is the equation of this line? _____

PreCalculus Notes – Section 1.9

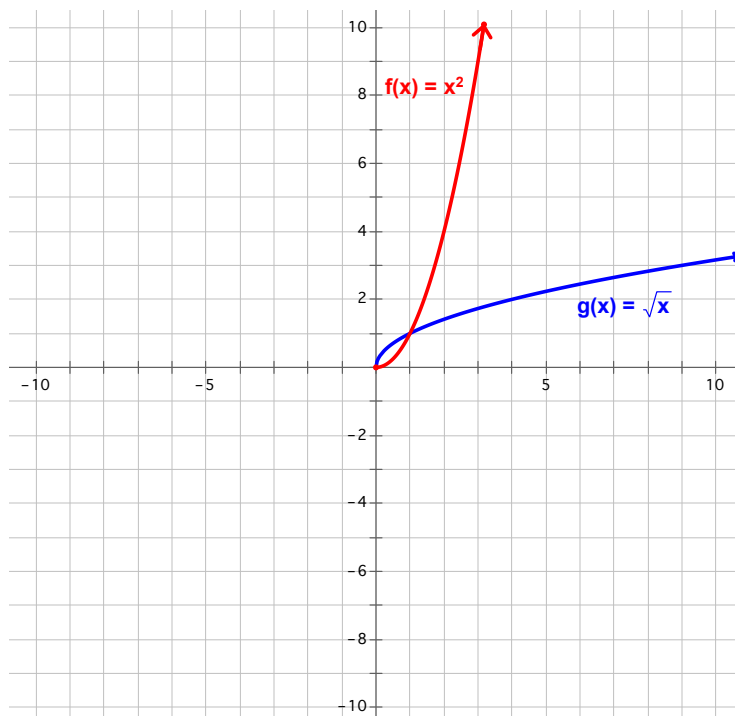
Ex 2) Sketch the graphs of the inverse functions $f(x) = x^2$ ($x \geq 0$) and $f^{-1}(x) = \sqrt{x}$ on the same set of axes and show that the graphs are reflections of each other in the line $y = x$.

$$f(x) = x^2$$

$$f^{-1}(x) = \sqrt{x}$$

x	0	1	2	3
y	0	1	4	9

x	0	1	4	9
y	0	1	2	3



Finding an Inverse Function Algebraically

To find an inverse function algebraically, we switch x and y and solve for y !

Ex. 3) Find the inverse function of $f(x) = \frac{x+3}{x-2}$, algebraically.

$$y = \frac{x+3}{x-2} \rightarrow x = \frac{y+3}{y-2}$$

$$\Rightarrow x(y-2) = y+3$$

$$\Rightarrow xy - 2x = y+3$$

$$\Rightarrow xy - y = 2x+3$$

$$\Rightarrow y(x-1) = 2x+3$$

$$\Rightarrow y = \frac{2x+3}{x-1}$$

Try this!

Ex 4) Find the inverse function of $f(x) = \frac{5-3x}{2}$, algebraically.

Verifying Inverse Functions Algebraically

Let f and g be two functions such that $f(g(x)) = x$ for every x in the domain of g , and $g(f(x)) = x$ for every x in the domain of f .

Then $g(x)$ is the **inverse function** of $f(x)$ and $g(x)$ can be denoted by $f^{-1}(x)$.

Examples

Verify that $f(x)$ and $g(x)$ are inverse functions algebraically

a. $f(x) = 3 - 4x$ $g(x) = \frac{3-x}{4}$

1st: Find $f(g(x))$:

$$\begin{aligned} f(g(x)) &= 3 - 4\left(\frac{3-x}{4}\right) \\ &= 3 - 1(3-x) \\ &= 3 - 3 + x \\ &= x \end{aligned}$$

2nd: Find $g(f(x))$:

Try this!!

b. Verify that $f(x)$ and $g(x)$ are inverse functions algebraically (in one direction only)

$$f(x) = \frac{x+1}{x-2} \quad g(x) = \frac{2x+1}{x-1}$$

$$\begin{aligned} f(g(x)) &= \frac{\left(\frac{2x+1}{x-1}\right)+1}{\left(\frac{2x+1}{x-1}\right)-2} \\ &= \frac{(2x+1)+1(x-1)}{(2x+1)-2(x-1)} \\ &= \frac{3x}{2x+1-2x+2} \\ &= \frac{3x}{3} \\ &= x \end{aligned}$$

One-to-One Functions

A function is “one-to-one” if for every y there is exactly one x .

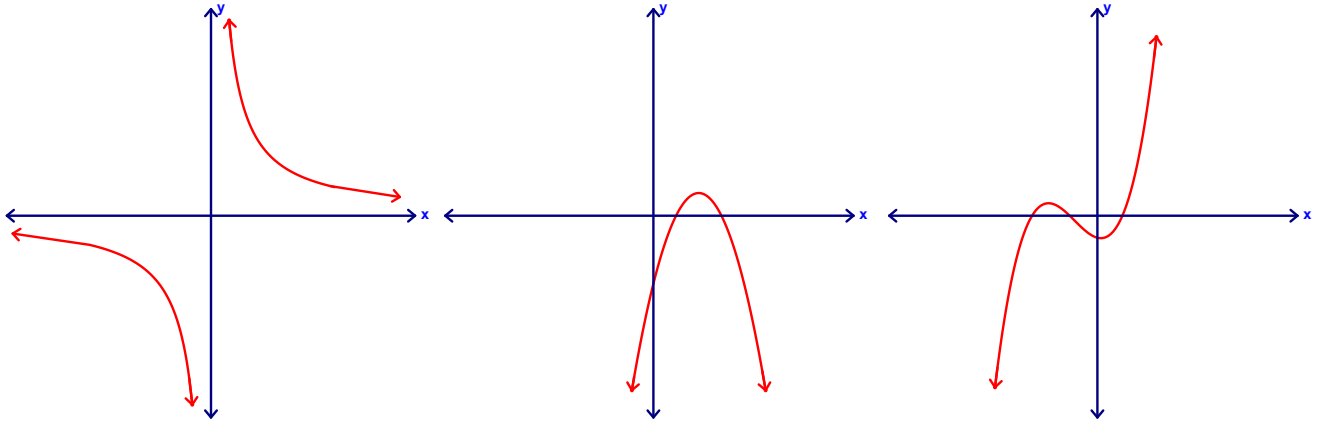
A quick way to tell is by applying the **horizontal** line test.

Horizontal Line Test

A function f has an inverse function iff no **horizontal** line intersects the graph of f at **more than one** point.

→ A function f has an inverse function iff f is **one-to-one**.

Examples



a.

b.

c.

One to one? _____ One to one? _____ One to one? _____

Has inverse? _____ Has inverse? _____ Has inverse? _____

Suggested Practice Problems: p.99-101 (these are all odds and answers in back of the book)
#1, 3, 13-23 (part a), 25, 29, 31, 39, 43, 45, 47, 51, 55, 57, 65, 67