Section 2.1 – Quadratic Functions

Definition of a Quadratic Function

- Let a, b, and c be real numbers with $a \neq 0$,

  The function $f(x)=ax^2+bx+c$ is called a QUADRATIC function in standard form.

- "a" is called the leading coefficient
  - If the leading coefficient is positive, the parabola opens upward (like a smile)
  - If the leading coefficient is negative, the parabola opens downward (like a frown)
- All parabolas are symmetric with respect to a line called the axis of symmetry
  (Or simply the *axis* of the parabola)
- The point where the axis intersects the parabola is the vertex of the parabola.

Vertex Form of a Quadratic Function

- The quadratic function $f(x)=a(x-h)^2+k$, $a\neq 0$ is in vertex form.
- The graph of f is a parabola whose axis is the vertical line x = h and whose vertex is (h,k).

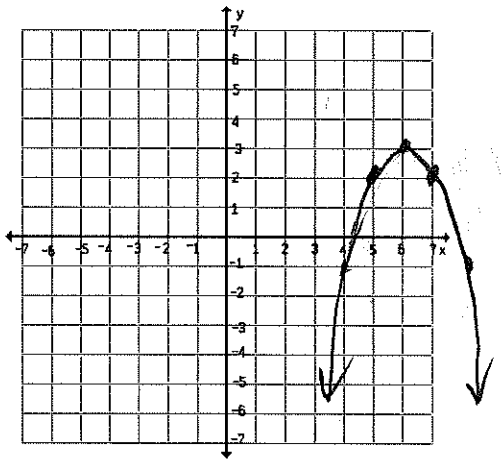
Examples

1) Sketch a graph of the parabola and identify the vertex and axis of symmetry of the parabola.

a. $f(x)=-(x-6)^2+3$

Vertex (6, 3)

axis $\Rightarrow x=6$



Section 2.1 – Quadratic Functions

If a quadratic function is in standard form, it is easier to find the vertex of the graph like this:

The vertex of $f(x)=ax^2+bx+c$ is (h,k), where $h=\frac{-b}{2a}$ and $k=f(h)$

2) Sketch a graph of the parabola and identify the vertex and x-intercepts of the parabola.

a) $g(x)=x^2+2x+1$

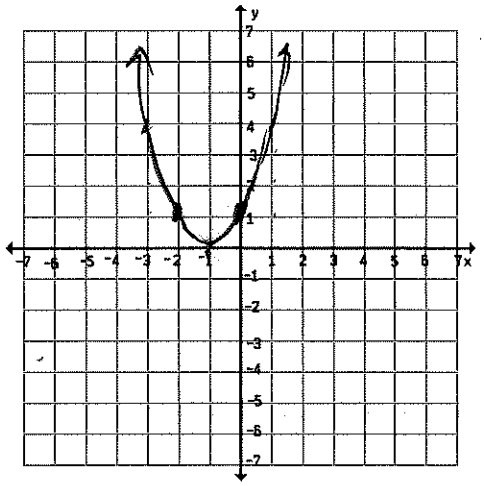
$\frac{-2}{2}=-1$

$g(-1)=(-1)^2+2(-1)+1=0$

vertex: $(-1,0)$

axis $\Rightarrow x=-1$

x-intercept = vertex = $(-1,0)$



b) $f(x)=x^2+2x-30$

$\frac{-2}{2}=-1$

$f(-1)=(-1)^2+2(-1)-30=-31$

vertex: $(-1,-31)$

axis $\Rightarrow x=-1$

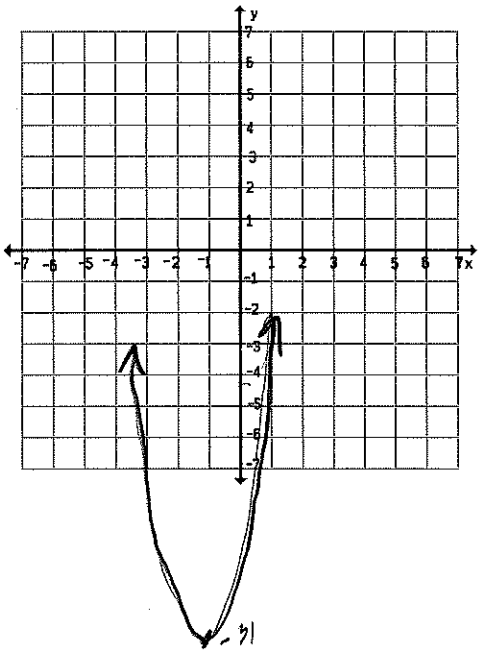
$0=x^2+2x-30$

$\Rightarrow \frac{-2\pm\sqrt{4-4(1)(-30)}}{2}$

$\Rightarrow -1\pm\sqrt{31}$

$(-1+\sqrt{31},0)$ ← x-intercepts

$(-1-\sqrt{31},0)$



Section 2.1 – Quadratic Functions

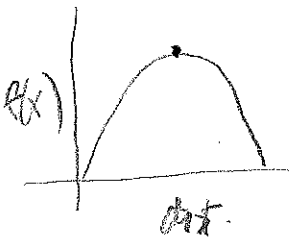
3) The Maximum Height of a Baseball

A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and at an angle of 45 degrees with respect to the ground. The path of the baseball is given by the function

$$f(x) = -0.0032x^2 + x + 3$$

where $f(x)$ is the height of the baseball in feet, and x is the horizontal distance from home plate in feet.

What is the maximum height reached by the baseball?



Max is vertex!

$\frac{-b}{2a} \Rightarrow \frac{-1}{2(-.0032)} = \frac{625}{4} = 156.25$

$f(156.25) = 81.125$ ft

4) Finding the Vertex of a Parabola by completing the square! (instead of using $h = \frac{-b}{2a}$)

(note: this method is easiest to use when a = 1)

a) $f(x) = x^2 + 8x + 11$

$= (x^2 + 8x + 16) + 11 - 16$

$= (x+4)^2 - 5$

Vertex: $(-4, -5)$

b) $f(x) = x^2 - 14x + 9$

$= (x - 14x + 49) + 9 - 49$

$= (x-7)^2 - 40$

Vertex: $(7, -40)$

Section 2.1 – Quadratic Functions

5) Writing the Equation of a Parabola

a) Write the vertex form of the equation of the parabola whose vertex is (1,2) and that passes through the point (0,0).
Sketch a graph too!

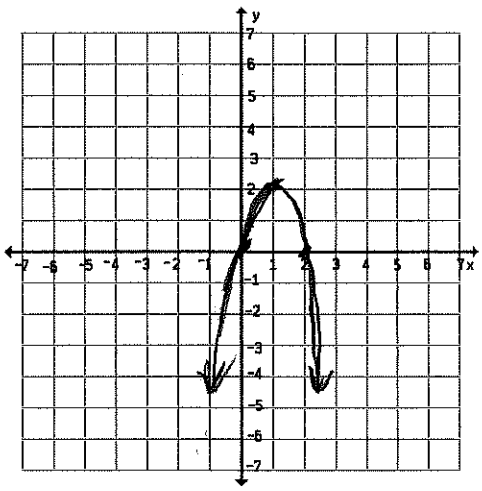
$f(x) = a(x-1)^2 + 2$

Plug in (0,0) → $0 = a(0-1)^2 + 2$

$-2 = a$

$\Downarrow$

$f(x) = -2(x-1)^2 + 2$



b) Write the vertex form of the equation of the parabola whose vertex is (4,-1) and that passes through the point (2,3).
Sketch a graph too!

$f(x) = a(x-4)^2 - 1$

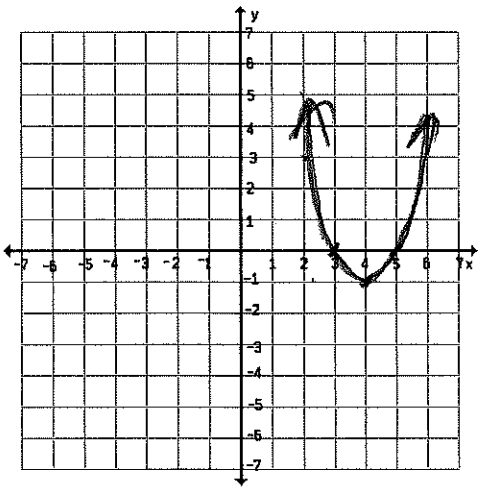
Plug in (2,3)

$3 = a(2-4)^2 - 1$

$4 = 4a$

$1 = a$

$f(x) = (x-4)^2 - 1$



Section 2.1 – Quadratic Functions

6) A study was done to compare the speed x (in miles per hour) with the mileage y (in miles per gallon) of an automobile. The results are shown in the table.

Speed, x	Mileage, y
15	22.3
20	25.5
25	27.5
30	29.0
35	28.8
40	30.0
45	29.9
50	30.2
55	30.4
60	28.8
65	27.4
70	25.3
75	23.3

a) Use your graphing calculator to create a scatter plot of the data
(STAT- EDIT—enter the data)

b) Use the regression feature of the calculator to find a quadratic model for the data
(STAT- CALC- QuadReg-ENTER)

$y = -.0081998002x^2 + .7461138861x + 13.46863137$

c) Enter the equation you just found into the "y =" so that it graphs the regression on top of the scatter plot. (OR place Y1 into "Store RegEQ")

d) Estimate the speed for which the mileage is the **greatest**.

45.495849 mph

(30.441177 mpg)

OR $\frac{-b}{2a} = \frac{-.7461138861}{2(-.0081998002)}$

≈ 45.4959

Homework Day 1: p.134 #1-8, 9, 13, 18, 24, 25, 37, 40, 44, 45, 53, 56, 65 (where it says "standard form" use "vertex form")

Homework Day 1 ½: p.136, #77,78,79