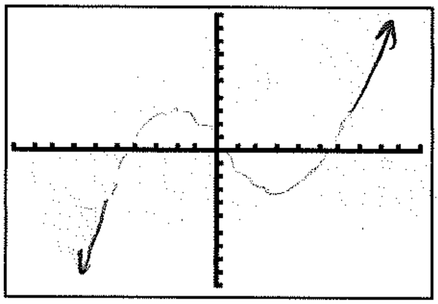
Section 2.2 – Polynomials of Higher Degree

For a graph of $y = x^n$,

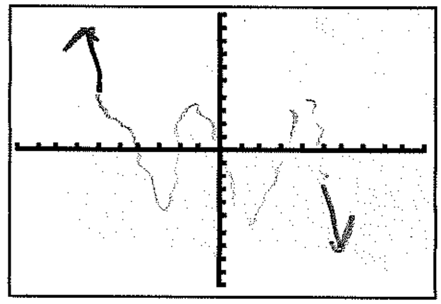
- if n is **even**, the graph touches (bounces off) the axis at the x-intercept.
- if n is **odd**, the graph crosses (passes through) the axis at the x-intercept.

The Leading Coefficient Test $\left(f(x) = a_n x^n + \ldots + a_1 x^1 + a_0\right)$

When n is odd.....

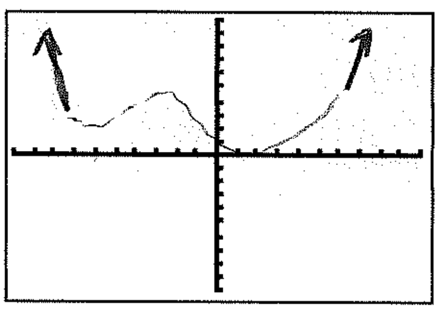


$a_n > 0$

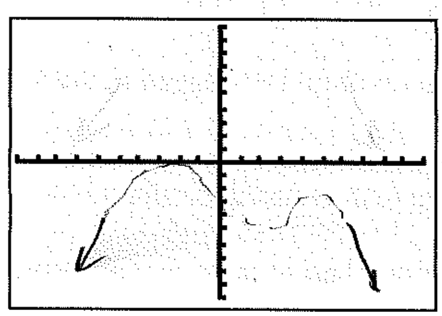


$a_n < 0$

When n is even.....



$a_n > 0$



$a_n < 0$

Section 2.2 – Polynomials of Higher Degree

Applying The Leading Coefficient Test

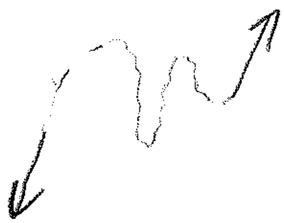
- This test is used to determine the right and left end behavior of the graph of the function.

Examples:

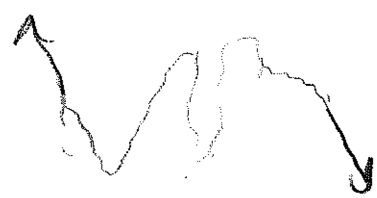
a) $f(x)=2x^2-3x+1$

b) $h(x)=1-x^6$

c) $f(x)=2x^5-5x+7.5$



d) $f(x)=-3x^7+2$



Zeros of Polynomial Functions

For a polynomial function f of **degree n**, the following statements are true:

- The graph of f has, at most, **n-1** turning points (points where the graph changes from increasing to decreasing or vice versa).
- The function f has, at most, **n** real zeros.
- If n is odd, the function f has at least one real zero.

Real Zeros of Polynomial Functions

If f is a polynomial function and a is a real number, the following statements are equivalent:

1) x = a is a ZERO of the function f.
2) x = a is a SOLUTION of the polynomial equation $f(x)=0$.
3) $(x-a)$ is a FACTOR of the polynomial $f(x)$.
4) $(a,0)$ is an X-INTERCEPT of the graph of f.

Section 2.2 – Polynomials of Higher Degree

Repeated Zeros

A factor $(x-a)^k$, $k > 1$, yields a repeated zero $x=a$ of multiplicity k.

1) If k is odd, the graph *crosses* the x-axis at $x=a$.
2) If k is even the graph *touches* the x-axis at $x=a$.

Examples: Find the zeros of a polynomial function and determine the multiplicity of each zero.

1) $f(x)=x^3-4x^2+4x$

$= x(x^2-4x+4)$

$= x(x-2)^2$

$x=0$, $x=2$ multiplicity 2

2) $f(x)=x^4-x^3-20x^2$

$= x^2(x^2-x-20)$

$= x^2(x-5)(x+4)$

$x=0$, $x=5$, $x=-4$

multiplicity 2 multiplicity 1

3) $f(x)=x^3+x^2-6x$

$= x(x^2+x-6)$

$= x(x+3)(x-2)$

$x=0$, $x=-3$, $x=2$ all mult. 1

4) $f(x)=x^3-6x^2+9x$

$= x(x^2-6x+9)$

$= x(x-3)^2$

$x=0$, $x=3$ mult 2

Section 2.2 – Polynomials of Higher Degree

Graphs of Polynomial Functions

To sketch the graph of a polynomial function:
1) Apply the leading coefficient test.
2) Find the zeros of the polynomial.
3) Make a number line and test values between the zeros- this will determine whether your graph lies above or below the x-axis.
4) Draw the graph.

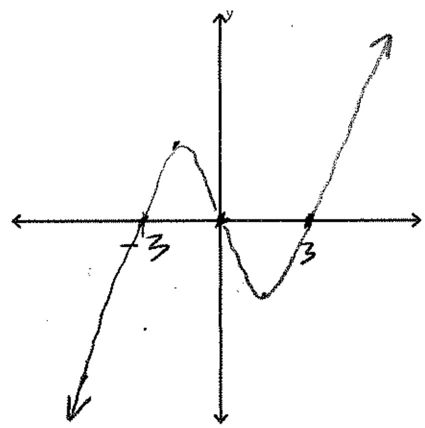
1) Sketch the graph of the polynomial functions.

a) $f(x)=x^3-9x$

$0=x(x^2-9)=x(x-3)(x+3)$

$x=0,\ x=3,\ x=-3$

multiplicity 1



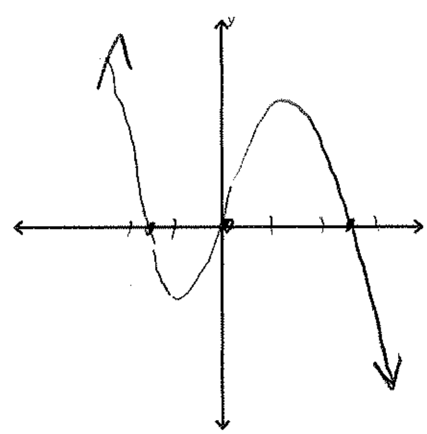
b) $f(x)=-4x^3+4x^2+15x$

$0=-x(4x^2-4x-15)$

$0=-x(2x+3)(2x-5)$

$x=0,\ x=-\frac{3}{2},\ x=\frac{5}{2}$

multiplicity 1



c) $f(x) = -2x^3 + 12x^2 - 18x$

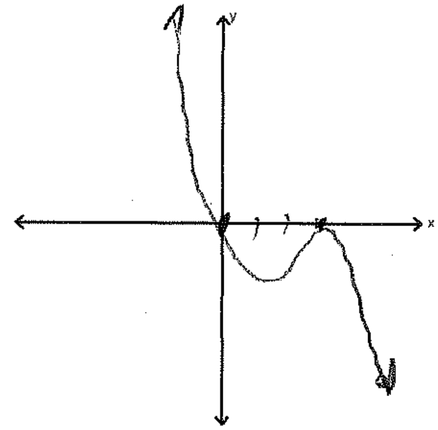
$0 = -2x(x^2 - 6x + 9)$

$0 = -2x(x-3)^2$

$x = 0,\ x = 3$

multiplicity 1

multiplicity 2



2) Find the equation in standard form of a polynomial of degree n that has the given zeros.

a) zeros: x = -8, -4
degree: n = 2

$(x+8)(x+4)$

$y = x^2 + 12x + 32$

b) zeros: x = 9
degree: n = 3

$(x-9)^3 = (x-9)(x^2 - 18x + 81)$

$= x^3 - 18x^2 + 81x - 9x^2 + 162x - 729$

$y = x^3 - 27x^2 + 243x - 729$

c) zeros: $x = 2, 4 + \sqrt{5}, 4 - \sqrt{5}$
degree: n = 4

$(4+\sqrt{5})(4-\sqrt{5})$

$-4x - \sqrt{5}x - 4x + \sqrt{5}x$

$y = (x-2)^2(x-(4+\sqrt{5}))(x-(4-\sqrt{5}))$

$= (x^2 - 4x + 4)(x - 4 - \sqrt{5})(x - 4 + \sqrt{5})$

$= (x^2 - 4x + 4)(x^2 - 8x + 16 - 5)$

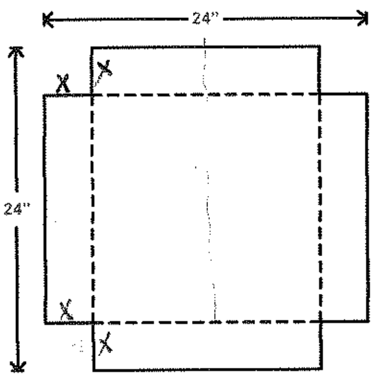
$= (x^2 - 4x + 4)(x^2 - 8x + 11)$

$= \cancel{x^4} - \cancel{8x^3} + \cancel{11x^2} - \cancel{4x^3} + \cancel{32x^2} - \cancel{44x} + \cancel{4x^2} - 32x + 44$

$y = x^4 - 12x^3 + 47x^2 - 76x + 44$

Section 2.2 – Polynomials of Higher Degree

3) An open box with locking tabs is to be made from a square piece of material 24 inches on each side, this is to be done by cutting equal squares from the corners and folding along dashed lines shown in the figure.



a) What is the volume of the box in terms of x?

$V(x) = BH = (24-2x)^2 x = (576 - 96x + 4x^2)x$
$= 4x^3 - 96x^2 + 576x$

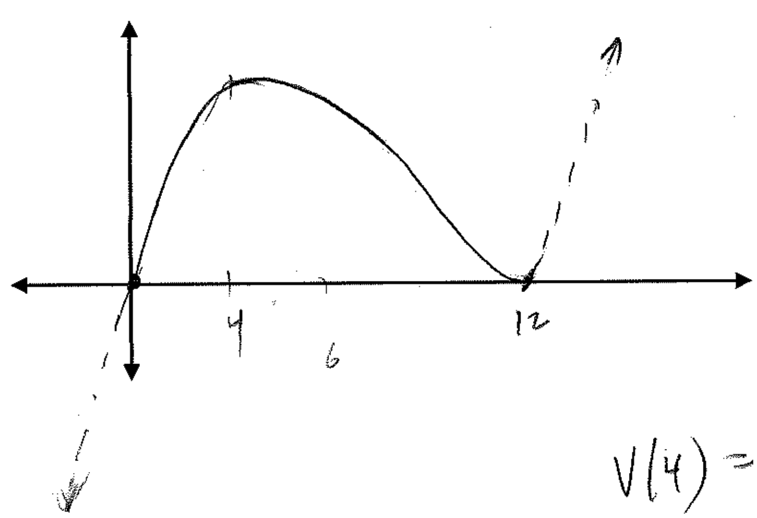
$24 - 2x = 0$
$24 = 2x$
$12 = x$

b) What is the domain of the function V?

$\{x \mid 0 < x < 12\}$

0, 12
multiplicity 2
multiplicity 1

c) Sketch a graph of the function and find the value of x that will give the maximum volume.



$V(4) = 1{,}024$

HW: p. 148-149 #1-8 all, 13-21 odd, 27-30 all.

HW: p. 148-150 #9, 11, 43, 44, 47-49, 58, 59, 67, 70, 71