Section 2.3 – Long/Synthetic Division and the Remainder & Factor Theorems

Long Division of Polynomials

Polynomial Long Division is a lot like dividing integers (you should all be experts at this! ☺)

$$
\begin{array}{r}
112 \\
32\overline{)3587} \\
32 \\
\hline
38 \\
32 \\
\hline
67 \\
64 \\
\hline
3
\end{array}
$$

remainder: 3

Now let's try it with polynomials instead of ordinary numbers:

Ex.1)

$$
\begin{array}{r}
x^2+5x+6 \\
x-2\overline{)x^3+3x^2-4x-12} \\
-(x^3-2x^2) \\
\hline
5x^2-4x \\
5x^2-10x \\
\hline
6x-12 \\
6x-12 \\
\hline
0
\end{array}
$$

Ex.2) Divide $2x^4+3x^3+5x-1$ by x^2-2x+2 (careful setting this one up!)

$$
\begin{array}{r}
2x^2+7x+10 \\
x^2-2x+2\overline{)2x^4+3x^3+0x^2+5x-1} \\
-(2x^4-4x^3+4x^2) \\
\hline
7x^3-4x^2+5x \\
-(7x^3-14x^2+14x) \\
\hline
10x^2-9x-1 \\
-(10x^2-20x+20) \\
\hline
11x-21
\end{array}
$$

← remainder

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Synthetic Division of Polynomials

Synthetic division can be used to divide a polynomial by an expression of the form x-k.

Let's do Example 1 again, but this time with synthetic division

$$x-2\overline{\big)x^3+3x^2-4x-12}$$

In synthetic division, you don't write the variables.

Step 1: Write the coefficients of the polynomial and then write the k–value (2) of the divisor $x-2$ on the left. Write the 1st coefficient 1 below the line.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$

Step 2: Multiply the k–value (2) by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & & \\ \hline & 1 & & & \end{array}$$

Step 3: Write the sum of 3 and 2 below the line. Multiply 2 by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & \\ \hline & 1 & 5 & & \end{array}$$

Step 4: Write the sum of -4 and 10 below the line. Multiply 2 by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & 0 \leftarrow \text{remainder} \end{array}$$

The remainder is 0, and the resulting numbers 1, 5, and 6 are the coefficients of the quotient

So your answer is...... x^2+5x+6 Just like with long division!

Let's try some more:

Ex 3) $(5x^4-2x^3+7x^2+6x-8)\div(x-4)$

$$\begin{array}{r|rrrrr} 4 & 5 & -2 & 7 & 6 & -8 \\ & & 20 & 72 & 316 & 1288 \\ \hline & 5 & 18 & 79 & 322 & 1280 \end{array}$$ remainder

$$\frac{5x^4-2x^3+7x^2+6x-8}{x-4} = \boxed{5x^4+18x^2+79x+322+\frac{1280}{x-4}}$$

Ex 4) Divide $x^3-10x-24$ by $x+2$ (be careful...)

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -10 & -24 \\ & & -2 & 4 & 12 \\ \hline & 1 & -2 & -6 & -12 \end{array} \leftarrow \text{remainder}$$

$$x^3-10x-24 \div x+2 = \boxed{x^3-2x^2-6x-\frac{12}{x+2}}$$

Now YOU try:

Ex 5) $(x^2+7x+4)\div(x+3)$

$$\begin{array}{r|rrr} -3 & 1 & 7 & 4 \\ & & -3 & -12 \\ \hline & 1 & 4 & -8 \end{array} \leftarrow \text{remainder}$$

$$\frac{x^2+7x+4}{x+3} = \boxed{x+4-\frac{8}{x+3}}$$

Ex 6) $(2x^3-3x+4)\div(x-1)$

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -3 & 4 \\ & & 2 & 2 & -1 \\ \hline & 2 & 2 & -1 & 3 \end{array}$$

$$\frac{2x^3-3x+4}{x-1} = \boxed{2x^2+2x-1+\frac{3}{x-1}}$$

The Remainder Theorem

An important case of the division algorithm occurs when the divisor is of the form $d(x)=x-k$.

Division Algorithm:

$f(x)=(x-k)\cdot q(x)+r$

↑ ↑ ↑ ↑

dividend divisor quotient remainder

When $x=k$, what does $f(x)$ equal?

$f(x)=r$ ✓

Ex 7) Let $f(x)=3x^3-2x^2+2x-5$

Divide $f(x)$ by $x-2$ using synthetic division. What is the remainder? How is $f(2)$ related to the remainder?

$$\begin{array}{r|rrrr} 2 & 3 & -2 & 2 & -5 \\ & & 6 & 8 & 20 \\ \hline & 3 & 4 & 10 & 15 \end{array} \leftarrow \text{remainder}$$

$f(2)=3(2)^3-2(2)^2+2(2)-5$!!

$=24-8+4-5=15$

Same!!

The Remainder Theorem:

If a polynomial $f(x)$ is divided by $x-k$, then the remainder is $r=f(k)$.

Ex 8) Find the remainder when $f(x)=3x^2+7x-20$ is divided by:

a) $x-2$

$$\begin{array}{r|rrr} 2 & 3 & 7 & -20 \\ & & 6 & 26 \\ \hline & 3 & 13 & 6 \end{array}$$

b) $x+4$

$$\begin{array}{r|rrr} -4 & 3 & 7 & -20 \\ & & -12 & 20 \\ \hline & 3 & -5 & 0 \end{array}$$

Because the remainder in part (b) is 0, $x+4$ **divides evenly** into $f(x)=3x^2+7x-20$.

So we know:

- $x+4$ is a FACTOR of $f(x)=3x^2+7x-20$,
- -4 is a ZERO of $3x^2+7x-20=0$, and
- $(-4,0)$ is an X-INTERCEPT of the graph of $y=3x^2+7x-20$.

We know all of this without ever dividing, factoring, or graphing.

The Factor Theorem:

A polynomial $f(x)$ has a factor $x-k$ if $f(k)=0$.

Ex 9) Factor $f(x)=2x^3+11x^2+18x+9$ given that $f(-3)=0$.

Because $f(-3)=0$, we know that <u>x + 3</u> is a factor of $f(x)$.
Use synthetic division to simplify the polynomial and find the other factors.

$$\begin{array}{r|rrrr} -3 & 2 & 11 & 18 & 9 \\ & & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$f(x)=(x+3)(2x^2+5x+3)$
$=(x+3)(2x+3)(x+1)$

Ex 10) One zero of $f(x)=x^3-2x^2-9x+18$ is $x=2$. Find the other zeros of the function.

Because $f(2)=0$, you know that <u>x-2</u> is a factor of $f(x)$.
Again, use synthetic division to factor completely.

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -9 & 18 \\ & & 2 & 0 & -18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$f(x)=(x-2)(x^2-9)$
$=(x-2)(x+3)(x-3)$

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Ex. 11) Use synthetic division to find the function value. Then verify your answer using substitution:

$g(x)=x^6-4x^4+3x^2+2,\ \ g(2)$

$$\begin{array}{r|rrrrrrr} 2 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & 2 & 4 & 0 & 0 & 6 & 12 \\ \hline & 1 & 2 & 0 & 0 & 3 & 6 & 14 \end{array}$$

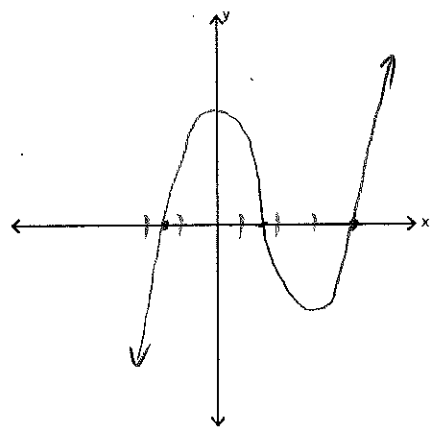
$$g(2) = 2^6-4(2)^4+3(2)^2+2 = 64-64+12+2 = 14$$

Ex 12) Use $f(x)=x^3-4x^2-2x+8$ to answer the following questions:

a) Use the zero feature on your calculator to approximate the zeros of the function.

My zeros are approximately -1.414, 1.414, and 4

b) Sketch:



c) Determine one of the exact zeros.

One of the exact zeros is 4

d) Use synthetic division to verify your result, and then factor the polynomial completely.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$f(x) = (x-4)(x^2-2) = \boxed{(x-4)(x+\sqrt{2})(x-\sqrt{2})}$$

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Homework: Page 159-160 #7, 9, 15, 21, 23, 27, 29, 35, 45(a&b), 49, 51, 65