Section 2.3 – Long/Synthetic Division and the Remainder & Factor Theorems

Long Division of Polynomials

Polynomial Long Division is a lot like dividing integers (you should all be experts at this! ☺)

$$32\overline{)3587}$$

Now let's try it with polynomials instead of ordinary numbers:

Ex.1)

$$x-2\overline{)x^3+3x^2-4x-12}$$

Ex.2) Divide $2x^4+3x^3+5x-1$ by x^2-2x+2 (careful setting this one up!)

Section 2.3 – Long/Synthetic Division and the Remainder & Factor Theorems

Synthetic Division of Polynomials

Synthetic division can be used to divide a polynomial by an expression of the form x-k.

Let's do Example 1 again, but this time with synthetic division

$$x-2\overline{\smash{)}x^3+3x^2-4x-12}$$

In synthetic division, you don't write the variables.

Step 1: Write the coefficients of the polynomial and then write the k-value (2) of the divisor $x-2$ on the left. Write the 1st coefficient 1 below the line.

$$\begin{array}{c|cccc} 2 & 1 & 3 & -4 & -12 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$

Step 2: Multiply the k–value (2) by the number below the line and write the product below the next coefficient.

$$\begin{array}{c|cccc} 2 & 1 & 3 & -4 & -12 \\ & & 2 & & \\ \hline & 1 & & & \end{array}$$

Step 3: Write the sum of 3 and 2 below the line. Multiply 2 by the number below the line and write the product below the next coefficient.

$$\begin{array}{c|cccc} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & \\ \hline & 1 & 5 & & \end{array}$$

Step 4: Write the sum of -4 and 10 below the line. Multiply 2 by the number below the line and write the product below the next coefficient.

$$\begin{array}{c|cccc} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & 0 \leftarrow \text{remainder} \end{array}$$

The remainder is 0, and the resulting numbers 1, 5, and 6 are the coefficients of the quotient

So your answer is…… x^2+5x+6 Just like with long division!

Let's try some more:

Ex 3) $(5x^4-2x^3+7x^2+6x-8)\div(x-4)$

Ex 4) Divide $x^3-10x-24$ by $x+2$ (be careful…)

Now YOU try:

Ex 5) $(x^2+7x+4)\div(x+3)$

Ex 6) $(2x^3-3x+4)\div(x-1)$

The Remainder Theorem

An important case of the division algorithm occurs when the divisor is of the form $d(x)=x-k$.

Division Algorithm:

$$f(x)=(x-k)\bullet q(x)+r$$

↑ dividend ↑ divisor ↑ quotient ↑ remainder

When $x=k$, what does $f(x)$ equal?

Section 2.3 – Long/Synthetic Division and the Remainder & Factor Theorems

Ex 7) Let $f(x)=3x^3-2x^2+2x-5$

Divide $f(x)$ by $x-2$ using synthetic division. What is the remainder? How is $f(2)$ related to the remainder?

<u>The Remainder Theorem:</u>

If a polynomial $f(x)$ is divided by $x-k$, then the remainder is $r=f(k)$.

Ex 8) Find the <u>remainder</u> when $f(x)=3x^2+7x-20$ is divided by:

a) $x-2$

b) $x+4$

Because the remainder in part (b) is 0, $x+4$ **divides evenly** into $f(x)=3x^2+7x-20$.

So we know:

- $x+4$ is a FACTOR of $f(x)=3x^2+7x-20$,
- -4 is a ZERO of $3x^2+7x-20=0$, and
- $(-4,0)$ is an X-INTERCEPT of the graph of $y=3x^2+7x-20$.

We know all of this without ever dividing, factoring, or graphing.

Section 2.3 – Long/Synthetic Division and the Remainder & Factor Theorems

The Factor Theorem:

A polynomial $f(x)$ has a factor $x-k$ if $f(k)=0$.

Ex 9) Factor $f(x)=2x^3+11x^2+18x+9$ given that $f(-3)=0$.

Because $f(-3)=0$, we know that ______________ is a factor of $f(x)$.
Use synthetic division to simplify the polynomial and find the other factors.

Ex 10) One zero of $f(x)=x^3-2x^2-9x+18$ is $x=2$. Find the other zeros of the function.

Because $f(2)=0$, you know that _________ is a factor of $f(x)$.
Again, use synthetic division to factor completely.

Section 2.3 – Long/Synthetic Division and the Remainder & Factor Theorems

Ex. 11) Use synthetic division to find the function value. Then verify your answer using substitution:

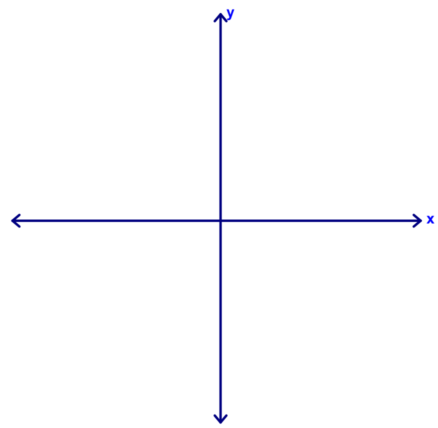
$g(x) = x^6 - 4x^4 + 3x^2 + 2, \quad g(2)$

Ex 12) Use $f(x) = x^3 - 4x^2 - 2x + 8$ to answer the following questions:

a) Use the zero feature on your calculator to approximate the zeros of the function.

My zeros are approximately _________, ________, and ___________

b) Sketch:



c) Determine one of the exact zeros.

One of the exact zeros is __________

d) Use synthetic division to verify your result, and then factor the polynomial completely.

Homework: Page 159-160 #7, 9, 15, 21, 23, 27, 29, 35, 45(a&b), 49, 51, 65