KEY

Section 2.5 – Zeros of Polynomial Functions

The polynomial function $f(x)=63x^3-129x^2+28x+20$ has $x=\frac{5}{3},-\frac{2}{7},\frac{2}{3}$ as its zeros.

Notice that the <u>NUMERATORS</u> of these zeros (5, -2, and 2) are factors of the *constant term*, 20.

Also notice that the <u>DENOMINATORS</u> (3 and 7) are factors of the *leading coefficient*, 63.

<u>The Rational Zero Theorem</u>

If $f(x)=a_nx^n+...+a_1x^1+a_0$ has integer coefficients, then *every* rational zero of $f(x)$ has the following form:

$$\frac{p}{q}=\frac{\text{factors of constant term}}{\text{factors of leading coefficient}}$$

Ex 1) Find the rational zeros of $f(x)=x^3+2x^2-11x-12$

1st: <u>List</u> the possible rational zeros

constant term's factors: 1, 2, 3, 4, 6, 12, -12, -6, -4, -3, -2, -1

leading coefficient's factors: 1, -1

So, the possible rational zeros are:

x = $\pm\frac{12}{1}, \pm\frac{6}{1}, \pm\frac{4}{1}, \pm\frac{2}{1}, \pm\frac{1}{1}$ x = 12, 6, 4, 3, 2, 1, -1, -2, -3, -4, -6, -12

2nd: <u>Test</u> these zeros using synthetic division

Try x = 1

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -11 & -12 \\ & & 1 & 3 & -8 \\ \hline & 1 & 3 & -8 & -20 \end{array}$$

NO

Try x = -1

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -11 & -12 \\ & & -1 & -1 & 12 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

yes!

Since −1 is a zero of $f(x)$, you can write the following: $f(x)$ = $(x+1)(x^2+x-12)$

Factor the trinomial and write the function in fully factored form:

$f(x)$ = $(x+1)(x+4)(x-3)$ so the zeros are: x = −1, −4, 3

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Ex 2) Find the rational zeros of $f(x) = 2x^3 - 3x^2 - 8x - 3$

1st: List the possible rational zeros

const. $\pm 1, \pm 3$

coeff $\pm 1, \pm 2$

The possible rational zeros are:

$x = \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{1}{1}, \pm\frac{1}{2} \rightarrow$ $x = 3, -3, 3/2, -3/2, 1/2, -1/2, 1, -1$

2nd: Test these zeros using synthetic division

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -8 & -3 \\ & & 6 & +9 & 3 \\ \hline & 2 & +3 & 1 & 0 \end{array}$$

yes!

$f(x) = (x-3)(2x^2+3x+1)$

Factor the trinomial and write the function in fully factored form:

$f(x) = (x-3)(2x+1)(x+1)$ so the zeros are: $x = 3, -1/2, -1$

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Ex 3) Use the following function $f(x)=2x^3+2x^2-8x-8$ to:

Find possible zeros, pick some and test them using synthetic division, factor completely. Then state all zeros.

$p: \pm 8, \pm 4, \pm 2, \pm 1 \quad > \pm 8, \pm 4, \pm 2, \pm 1, \pm \frac{1}{2}$

$q: \pm 2, \pm 1$

$$\begin{array}{r|rrrr} 2 & 2 & 2 & -8 & -8 \\ & & 4 & 12 & 8 \\ \hline & 2 & 6 & 4 & 0 \end{array} \quad \text{yes!}$$

$(x-2)(2x^2+6x+4) = (x-2)(2x+2)(x+2)$

$\boxed{x = 2, -1, -2}$

Ex 4) Use the following function $f(x)=x^4-x^3+x^2-3x-6$ to:

Find possible zeros, pick some and test them using synthetic division, factor completely. Then state all zeros.

$p: \pm 6, \pm 3, \pm 2, \pm 1 \quad > \pm 6, \pm 3, \pm 2, \pm 1$

$q: \pm 1$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 1 & -3 & -6 \\ & & -1 & 2 & -3 & 6 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array} \quad \text{yes!}$$

$(x+1)(x^3-2x^2+3x-6)$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 3 & -6 \\ & & 2 & 0 & 6 \\ \hline & 2 & 0 & 3 & 0 \end{array} \quad \text{yes!}$$

$(x+1)(x-2)(x^2+3)$

$\boxed{\text{Zeros: } -1, 2, \pm i\sqrt{3}}$

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What about when not all the zeros are REAL?

Conjugate Pairs Theorem: Imaginary zeros always come in conjugate pairs. Why?

Corollary to Conjugate Pairs Theorem:
A polynomial of odd degree must have at least one real zero. Why?

Example: Find **all** zeros of the polynomial $f(x)=3x^4+5x^3+25x^2+45x-18$

p: ±18, ±9, ±6, ±3, ±2, ±1 > ±18, ±9, ±6, ±3, ±2, ±1, ±2/3, ±1/3
q: ±3, ±1

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & 25 & 45 & -18 \\ & & -6 & 2 & -54 & 18 \\ \hline & 3 & -1 & 27 & -9 & 0 \end{array}$$ yes!

$(x+2)(3x^3-x^2+27x-9)$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & 27 & -9 \\ & & 1 & 0 & 9 \\ \hline & 3 & 0 & 27 & 0 \end{array}$$ yes!

$(x+2)(x-\frac{1}{3})(3x^2+27)$

$3(x+2)(x-\frac{1}{3})(x^2+9)$

$x=\pm 3i$

$\boxed{\text{zeros: } -2, \frac{1}{3}, 3i, -3i}$

Example: Find **all** zeros of the polynomial $f(x)=x^3+13x^2+57x+85$

p: ±85, ±17, ±5, ±1 > ±85, ±17, ±5, ±1
q: ±1

$$\begin{array}{r|rrrr} -5 & 1 & 13 & 57 & 85 \\ & & -5 & -40 & -85 \\ \hline & 1 & 8 & 17 & 0 \end{array}$$ yes!

$(x+5)(x^2+8x+17)$

$\dfrac{-8\pm\sqrt{64-4(1)(17)}}{2} = -4\pm\frac{2i}{2}$

$= -4\pm i$

$\boxed{\text{zeros: } -5, -4+i, -4-i}$

Homework: p. 179 #1, 3, 9, 11, ~~13, 23~~, 29, 65, 71