Section 2.5 – Zeros of Polynomial Functions

The polynomial function $f(x)=63x^3-129x^2+28x+20$ has $x=\frac{5}{3},-\frac{2}{7},\frac{2}{3}$ as its zeros.

Notice that the NUMERATORS of these zeros (5, -2, and 2) are factors of the *constant term*, 20.

Also notice that the DENOMINATORS (3 and 7) are factors of the *leading coefficient*, 63.

The Rational Zero Theorem

If $f(x)=a_n x^n+\ldots+a_1 x^1+a_0$ has integer coefficients, then *every* rational zero of $f(x)$ has the following form:

$$\frac{p}{q}=\frac{\text{factors of constant term}}{\text{factors of leading coefficient}}$$

Ex 1) Find the rational zeros of $f(x)=x^3+2x^2-11x-12$

1st: List the possible rational zeros

constant term's factors:

leading coefficient's factors:

So, the possible rational zeros are:

x = ____________________ → x = ______________

2nd: Test these zeros using synthetic division

Try x = 1 | Try x = -1

Since -1 is a zero of $f(x)$, you can write the following: $f(x)$ = ________________

Factor the trinomial and write the function in fully factored form:

$f(x)$ = ____________________ so the zeros are: x = _____, _______, ______

Section 2.5 – Zeros of Polynomial Functions

Ex 2) Find the rational zeros of $f(x) = 2x^3 - 3x^2 - 8x - 3$

1st: <u>List</u> the possible rational zeros

The possible rational zeros are:

x = ___________________________ $\rightarrow$ x = ___________________

2nd: <u>Test</u> these zeros using synthetic division

$f(x)$ = ___________________________

Factor the trinomial and write the function in fully factored form:

$f(x)$ = ___________________________ so the zeros are: x = ______, ________, _______

Section 2.5 – Zeros of Polynomial Functions

Ex 3) Use the following function $f(x) = 2x^3 + 2x^2 - 8x - 8$ to:

Find possible zeros, pick some and test them using synthetic division, factor completely. Then state all zeros.

Ex 4) Use the following function $f(x) = x^4 - x^3 + x^2 - 3x - 6$ to:

Find possible zeros, pick some and test them using synthetic division, factor completely. Then state all zeros.

Section 2.5 – Zeros of Polynomial Functions

What about when not all the zeros are REAL?

Conjugate Pairs Theorem: Imaginary zeros always come in conjugate pairs. Why?

Corollary to Conjugate Pairs Theorem:
A polynomial of odd degree must have at least one real zero. Why?

Example: Find **all** zeros of the polynomial $f(x)=3x^4+5x^3+25x^2+45x-18$

Example: Find **all** zeros of the polynomial $f(x)=x^3+13x^2+57x+85$

Homework: p. 179 #1, 3, 9, 11, 13, 23, 29, 65, 71