Section 2.7 – Solving Non-linear Inequalities – The Sign Chart

Let's start simply- solve the inequality:

$\boxed{\text{Ex 1}}$: Solve $3x-1\ge 0$ and answer using interval notation

$3x \geq 1$

$x \geq \frac{1}{3}$ $\boxed{[\frac{1}{3}, \infty)}$

Now, things get trickier when solving a non-linear inequality, like the quadratic one below.

$\boxed{\text{Ex 2}}$: Solve $x^2-4\ge 0$ and answer using interval notation

A) Try to do it with the same method as above where you isolate the variable first.

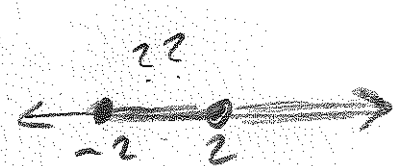
$x^2 \geq 4$

$x \geq 2$ but what about -2??

$x \geq \pm 2$? does that make sense?

B) Now use a graph to confirm (or deny) the answer

??



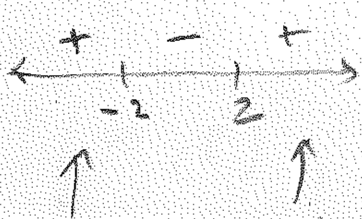
Check $x=3$ $3^2 \geq 4$

$x=-3$ $(-3)^2 \geq 4$ also true!

but not shaded on the graph

C) Solve it instead with the numerical/algebraic method called a sign chart (make sure to label it)

use $x = \pm 2$



both shaded

$(-\infty, -2] \cup [2, \infty)$

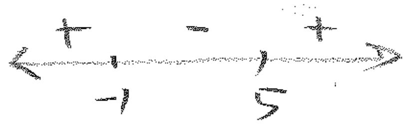
Section 2.7 – Solving Non-linear Inequalities – The Sign Chart

Ex 3: Use a sign chart to solve the quadratic inequality (hint: find your critical numbers first and use them to divide the number line)

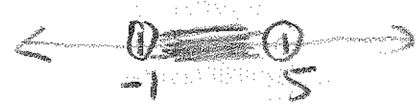
$x^2 - 4x - 5 < 0$

$(x-5)(x+1) < 0$

$x = 5 \quad x = -1$



we want < 0



$\boxed{(-1, 5)}$ interval notation for solution

Ex 4: Use a sign chart to solve the quadratic inequality (hint: NEVER divide both sides by a variable term…you will miss a critical number)

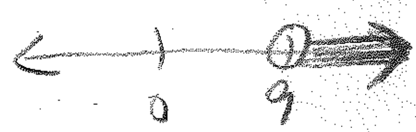
$x^3 > 9x^2$

$x^3 - 9x^2 > 0$

$x^2(x-9) > 0$

$x = 0 \quad x = 9$

want > 0



$\boxed{(9, \infty)}$

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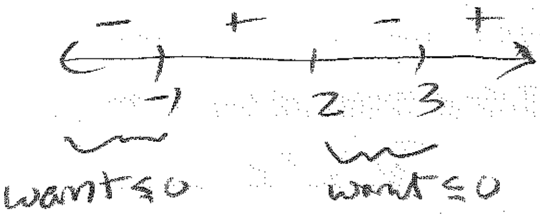
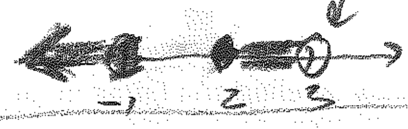
$\boxed{\text{Ex 5}}$: Use a sign chart to solve the rational inequality (hint: don't forget to also identify numbers where the denominator equals 0)

$$\frac{(x-2)(x+1)}{x-3} \le 0$$

$\left.\begin{array}{l} x=2,\ x=-1 \\ x=3 \end{array}\right]$ critical #'s



open b/c $x=3$ is not in domain



$\boxed{(-\infty,-1] \cup [2,3)}$

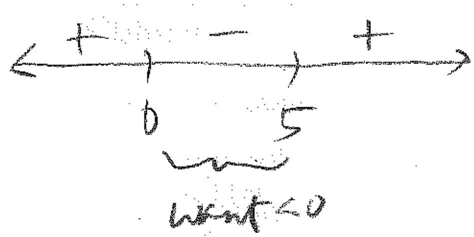
$\boxed{\text{Ex 6}}$: Solve:

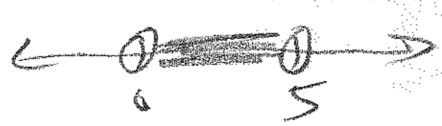
$x^4 - \cancel{} < 5x^3 + \cancel{x^2}$

$x^4 - 5x^3 < 0$

$x^3(x-5) < 0$

$x=0 \quad x=5$





$\boxed{(0,5)}$

Section 2.7 – Solving Non-linear Inequalities – The Sign Chart

Summary on How to Use a Sign Chart to solve a non-linear inequality

1) Draw a number line.
2) Find the zeros of the function AND the zeros of the denominator (if there is one) and use these critical numbers to split the number line into smaller intervals (these are the ONLY places where a sign change of $f(x)$ might occur!)
3) Pick a test x-value in each sub-interval and use it to determine whether $f(x)$ is positive or negative.
4) After completely labeling and filling in the sign chart, answer the question with appropriate notation.

How about this CHALLENGING one?

Solve: $\frac{2x-7}{x-5} \leq 3$

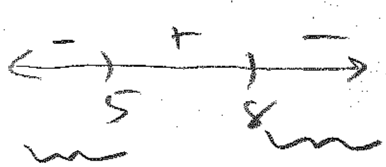
$$\frac{2x-7}{x-5} - 3 \leq 0$$

$$\frac{2x-7}{x-5} - \frac{3(x-5)}{x-5} \leq 0$$

$$\frac{2x-7-3x+15}{x-5} \leq 0$$

$$\frac{-x+8}{x-5} \leq 0$$

$$x=8, \quad x=5$$



$\boxed{(-\infty, 5) \cup [8, \infty)}$

Section 2.7 HW: p. 204 #9-14, 39, 47, 48