

Pre-Calculus CP 1 – Section 3.1 Notes
Exponential Functions and Their Graphs

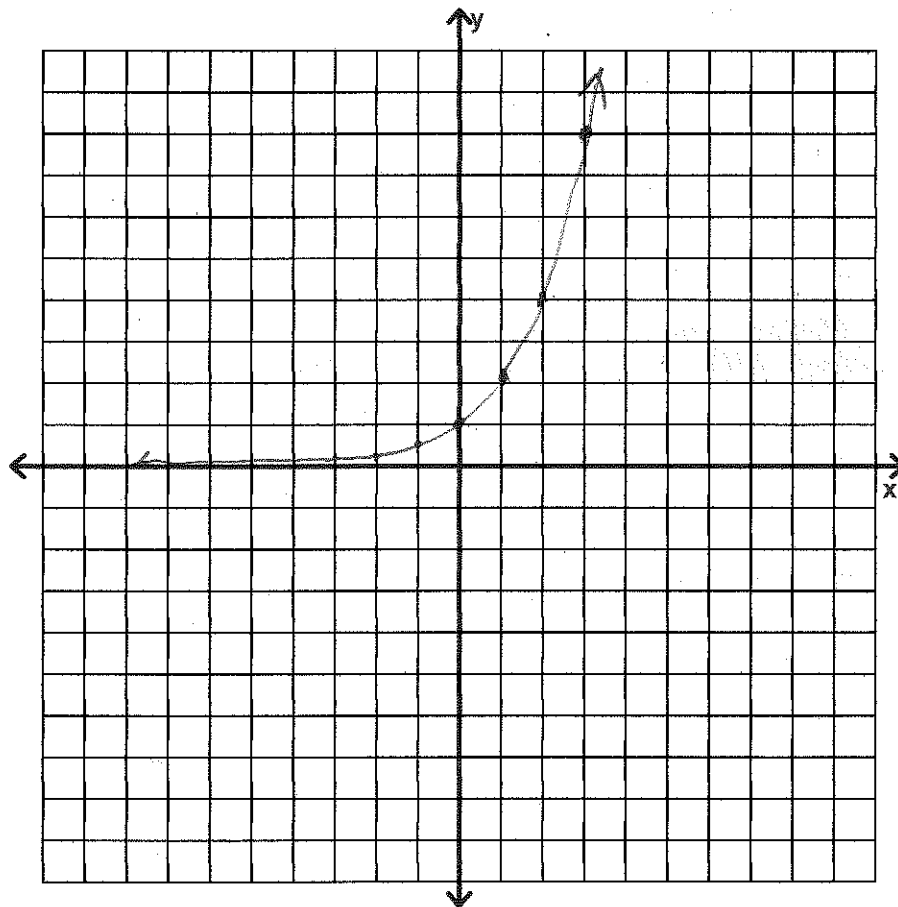
Name: KEY

Consider the functions $y = x^2$ and $y = 2^x$. Both functions have a base and an exponent. However, $y = x^2$ is a quadratic function (graph is a parabola), and $y = 2^x$ is an exponential function. Exponential functions have a fixed base and a variable for the exponent.

Exponential Function

The function $f(x) = b^x$ is an **exponential function** with **base** b , where b is a positive real number other than 1 and x is any real number.

x	$y = 2^x$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



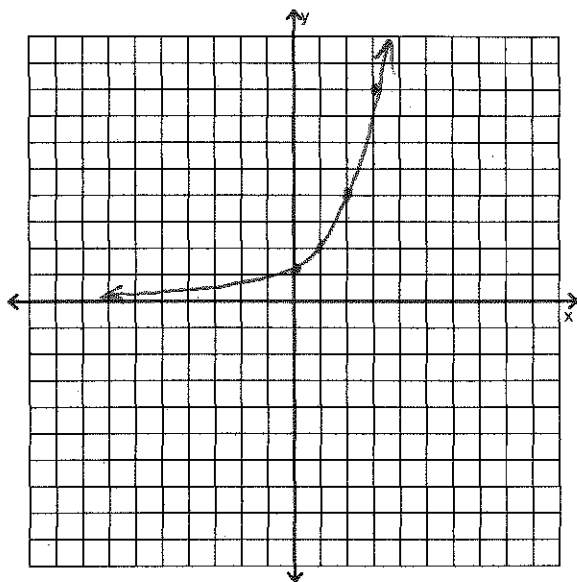
⇒ Is there an asymptote anywhere in this graph? Where?

$y = 0$

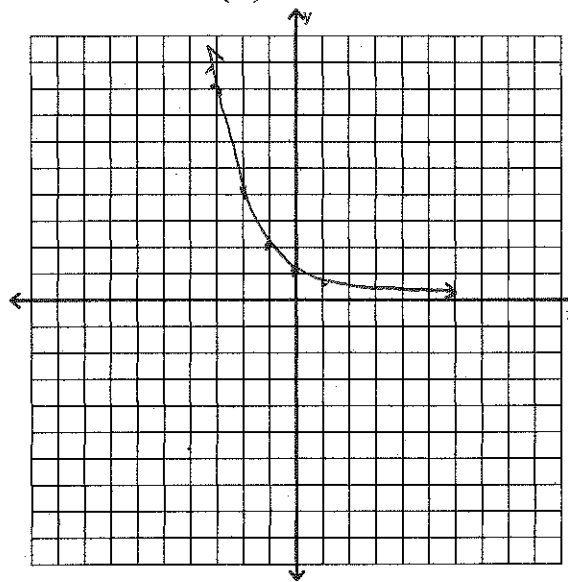
Exponential Functions and Their Graphs

The graphs below exhibit the two typical behaviors for exponential functions.

$$f(x) = 2^x$$



$$g(x) = \left(\frac{1}{2}\right)^x \text{ or } g(x) = 2^{-x}$$



When the multiplier (base) is greater than 1, the function displays **exponential growth** (getting bigger). When the multiplier is between 0 and 1, the function displays **exponential decay** (getting smaller).

Transformations

Use the graph of the parent function $f(x)$ to describe the transformation that yields the graph of $g(x)$:

Ex: $f(x) = 2^x$

$$g(x) = 2^{-x} + 1$$

reflect over y -axis
translate up 1

Ex: $f(x) = 4^x$

$$g(x) = \frac{1}{3} \left(\frac{1}{4}\right)^x$$

reflect over y -axis
vertical shrink by
factor of 3

Exponential Functions and Their Graphs

Try this: $f(x) = 6^x$

$$g(x) = -(6)^{x+5}$$

reflect over
x-axis
translate left 5

Try This: $f(x) = \left(\frac{1}{4}\right)^x$

$$g(x) = 4^x - 3$$

reflect over y-axis
down 3

One-to-One Property:

$$a^x = a^y \text{ if and only if } x = y$$

Solve the following for x:

Ex: $9 = 3^{x+1}$

$$3^2 = 3^{x+1}$$

$$2 = x + 1$$

$$x = 1$$

Ex: $\left(\frac{1}{2}\right)^x = 8$

$$2^{-x} = 2^3$$

$$-x = 3$$

$$x = -3$$

Try this: $5^x = \sqrt{125}$

$$5^x = 125^{\frac{1}{2}}$$

$$5^x = (5^3)^{\frac{1}{2}}$$

$$x = \frac{3}{2}$$

Try This: $81^x = \frac{1}{27}$

$$(3^4)^x = 3^{-3}$$

$$4x = -3$$

$$x = -\frac{3}{4}$$

Exponential Functions and Their Graphs

Investigation

Something interesting happens when $y = \left(1 + \frac{1}{x}\right)^x$ is found with increasing values of x

N	$\left(1 + \frac{1}{n}\right)^n$	Value, A
1	$\left(1 + \frac{1}{1}\right)^1$	2
4	$\left(1 + \frac{1}{4}\right)^4$	2.44140625
12	$\left(1 + \frac{1}{12}\right)^{12}$	2.61303529
365	$\left(1 + \frac{1}{365}\right)^{365}$	2.714567482
8760	$\left(1 + \frac{1}{8760}\right)^{8760}$	2.718126692
525,600	$\left(1 + \frac{1}{525,600}\right)^{525,600}$	2.718279243

We could summarize as $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x \approx 2.7182$

The number e

As n becomes very large, the value of $\left(1 + \frac{1}{n}\right)^n$ approaches the number

2.7182816..., which is named **e**.

e is an irrational number, like π : its decimal expansion continues forever without repeating.

e is also called "the natural base" and is used to estimate the ages of artifacts and to calculate interest that is compounded continuously – this is because the number **e** is a naturally occurring number that models the rate of continuous growth.

Exponential Functions and Their Graphs

Applications of Exponential Functions and the natural base e

Compounding Interest

After t years, the balance A in an account with principal P and annual interest r is given by the following formulas:

$$\text{For } n \text{ compounding per year: } A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$\text{For continuous compounding: } A = Pe^{rt}$$

Examples:

- 1) A deposit of \$5000 is made in a trust fund that pays 7.5% interest, compounded monthly. How much will be in the fund after the money has earned interest for 50 years?

$$A = 5000 \left(1 + \frac{0.075}{12} \right)^{(12)(50)} \approx \$210,138.69$$

How much is in the account if it is compounded continuously?

$$A = 5000 e^{(0.075)(50)} \approx \$212,605.4$$

- 2) The population P (in millions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-0.0039t}$, where t represents the year, with $t=6$ corresponding to 1996.

Is the population increasing or decreasing?

Find the population of Russia in 2000:

$$P = 152.26 e^{-0.0039(10)} \approx 146.44 \text{ million}$$

HW: Page 226-227 #'s 6-10, 17, 19, 21, 25, 31, 45-51, 61, 67