**Pre-Calculus CP 1 – Section 3.1 Notes** **Name:** ____________________

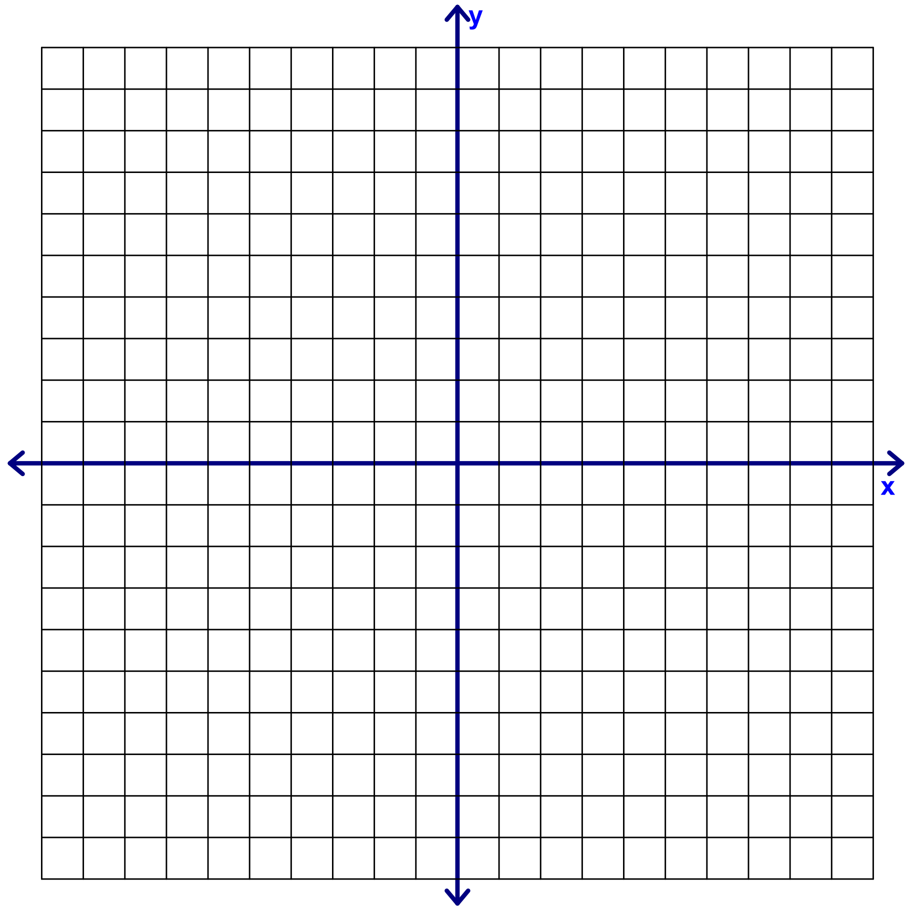
**Exponential Functions and Their Graphs**

Consider the functions $y = x^2$ and $y = 2^x$. Both functions have a base and an exponent. However, $y = x^2$ is a quadratic function (graph is a parabola), and $y = 2^x$ is an exponential function. Exponential functions have a fixed base and a variable for the exponent.

## Exponential Function

The function $f(x) = b^x$ is an **exponential function** with **base** b, where b is a positive real number other than 1 and x is any real number.

| x | $y = 2^x$ |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

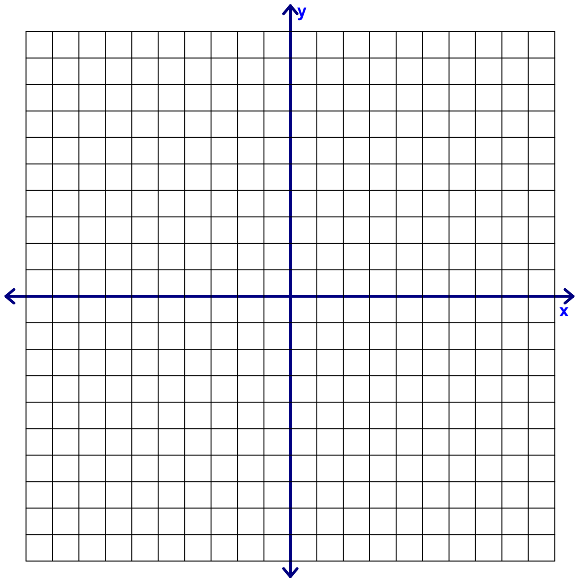



⇨ Is there an asymptote anywhere in this graph? Where?

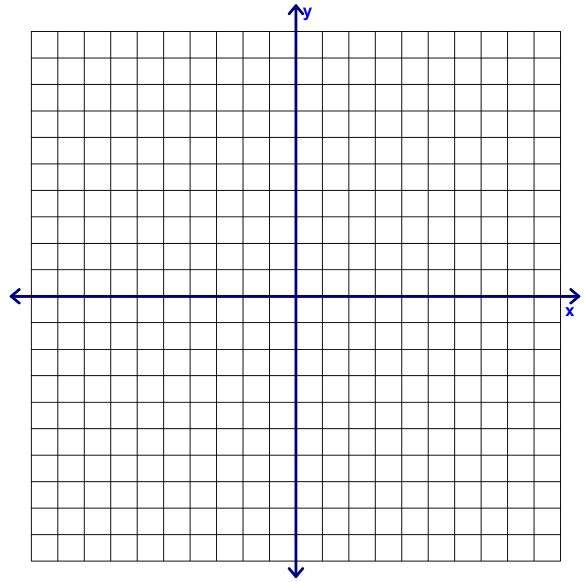
## Exponential Functions and Their Graphs

The graphs below exhibit the two typical behaviors for exponential functions.

$f(x) = 2^x$




$g(x) = \left(\frac{1}{2}\right)^x$ or $g(x) = 2^{-x}$




When the multiplier (base) is greater than 1, the function displays **exponential growth** (getting bigger). When the multiplier is between 0 and 1, the function displays **exponential decay** (getting smaller).

# Transformations

Use the graph of the parent function $f(x)$ to describe the transformation that yields the graph of $g(x)$:

Ex: $f(x) = 2^x$

$g(x) = 2^{-x} + 1$

Ex: $f(x) = 4^x$

$g(x) = \frac{1}{3}\left(\frac{1}{4}\right)^x$

## Exponential Functions and Their Graphs

Try this: $f(x)=6^x$

$g(x)=-(6)^{x+5}$

Try This: $f(x)=\left(\frac{1}{4}\right)^x$

$g(x)=4^x-3$

### One-to-One Property:

$a^x=a^y$ if and only if $x=y$

Solve the following for x:

Ex: $9=3^{x+1}$

Ex: $\left(\frac{1}{2}\right)^x=8$

Try this: $5^x=\sqrt{125}$

Try This: $81^x=\frac{1}{27}$

**Exponential Functions and Their Graphs**

## Investigation

Something interesting happens when $y=\left(1+\frac{1}{x}\right)^x$ is found with increasing values of x

| N | $\left(1+\frac{1}{n}\right)^n$ | Value, A |
|---|---|---|
| 1 | $\left(1+\frac{1}{1}\right)^1$ | 2 |
| 4 | $\left(1+\frac{1}{4}\right)^4$ | |
| 12 | $\left(1+\frac{1}{12}\right)^{12}$ | |
| 365 | $\left(1+\frac{1}{365}\right)^{365}$ | |
| 8760 | $\left(1+\frac{1}{8760}\right)^{8760}$ | |
| 525,600 | $\left(1+\frac{1}{525,600}\right)^{525,600}$ | |

We could summarize as $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \approx$

## The number e

As n becomes very large, the value of $\left(1+\frac{1}{n}\right)^n$ approaches the number 2.7182816…, which is named **e**.

**e** is an irrational number, like $\pi$ : its decimal expansion continues forever without repeating.

**e** is also called "the natural base" and is used to estimate the ages of artifacts and to calculate interest that is compounded continuously – this is because the number **e** is a naturally occurring number that models the rate of continuous growth.

## Applications of Exponential Functions and the natural base e

### Compounding Interest

After t years, the balance A in an account with principal P and annual interest r is given by the following formulas:

For n compounding per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

For continuous compounding: $A = Pe^{rt}$

***Examples:***

1) A deposit of \$5000 is made in a trust fund that pays 7.5% interest, compounded monthly. How much will be in the fund after the money has earned interest for 50 years?

   How much is in the account if it is compounded continuously?

2) The population P (in millions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-.0039t}$, where t represents the year, with t=6 corresponding to 1996.

   Is the population increasing or decreasing?

   Find the population of Russia in 2000:

HW: Page 226-227 #'s 6-10, 17, 19, 21, 25, 31, 45-51, 61, 67