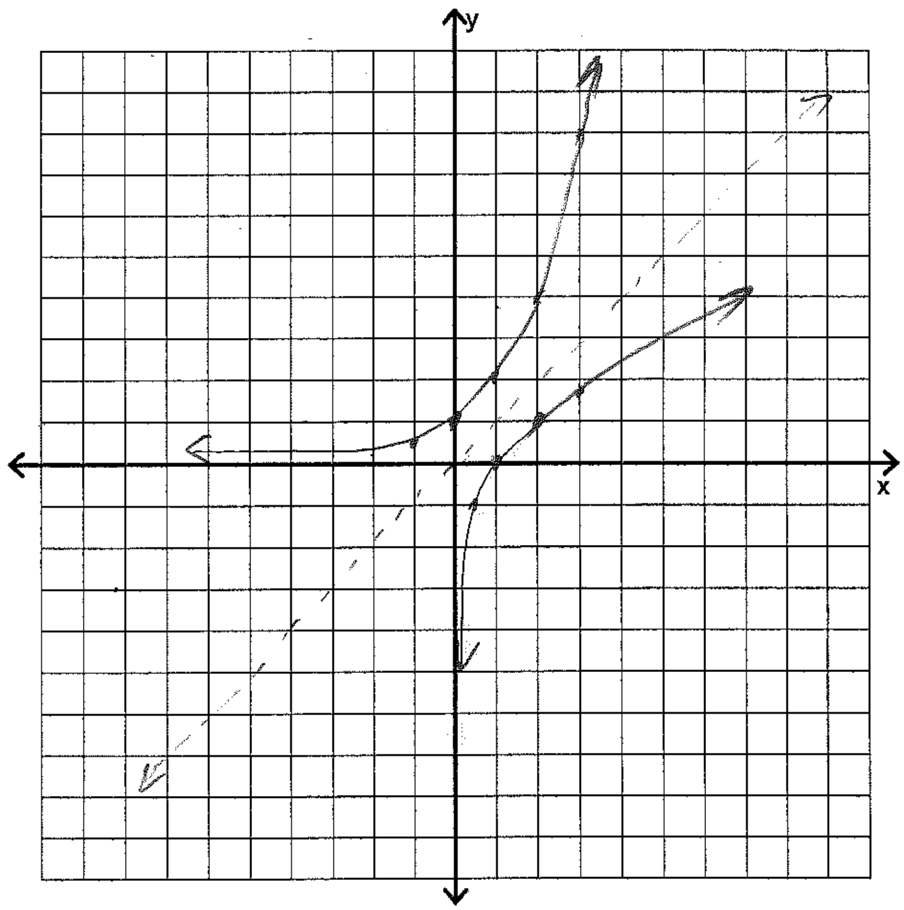
Pre-Calculus CP 1 – Section 3.2 Notes **Name:** KEY
Logarithmic Functions

Logarithmic Function

The function $f(x)=\log_b x$ is a **logarithmic function** with **base** b, where b is a positive real number and x is any real number.

x	$y=\log_2 x$
-3	DNE
-2	DNE
-1	DNE
0	DNE
1	0
2	1
3	1.58



⇨ Is there an asymptote anywhere in this graph? Where?

x=0

The common base for a logarithm is 10. Let's use this table of values to connect logs to exponentials:

x	-3	-2	-1	0	1	2	3
$y=10^x$	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000

Logarithmic Functions

A table of values for $y=10^x$ can be used to solve equations such as $10^x=1000$ and $10^x=\frac{1}{100}$.

However, to solve equations such as $10^x=85$ or $10^x=2.3$, a ***logarithm*** is needed. With logarithms, you can write an exponential equation in an ~~equivalent~~ logarithmic form because they are INVERSES of each other!

Equivalent Exponential and Logarithmic forms

For any positive base b, where $b \neq 1$:

$$y=b^x \leftrightarrow x=\log_b y$$

$y=b^x$
$x=b^y$
$y=\log_b x$

Ex: Write $5^3=125$ in logarithmic form.

$3=\log_5 125$

Ex: Write $\log_3 81=4$ in exponential form.

$3^4=81$

Find the missing parts in the table below:

Exponential form	$2^5=32$	$10^3=1000$	$3^{-2}=\frac{1}{9}$	$16^{\frac{1}{2}}=4$
Logarithmic form	$\log_2 32=5$	$\log_{10}1000=3$	$\log_3 \frac{1}{9}=-2$	$\log_{16}4=\frac{1}{2}$

You can evaluate logarithms with a base of 10 by using the LOG key on a calculator.

Ex: Solve for x: $10^x=85$

$\log_{10}85=x$
$x\approx 1.929$

Try This: $10^x=\frac{1}{109}$

$\log_{10}\frac{1}{109}=x$
$x\approx -2.037$

Logarithmic Functions

Fun Facts:

- The base-10 logarithm is called the **common logarithm**
  - The common logarithm, $\log_{10} x$, is usually written as "$\log x$"
- The base-e logarithm is called the **natural logarithmic function** and is denoted by the special symbol $\ln x$, read as "the natural log of x". Note the natural logarithm is also written without a base. The base is understood to be "e".

Properties of Logarithms

1. $\log_a 1 = 0$ because $a^0 = 1$
2. $\log_a a = 1$ because $a^1 = a$
3. $\log_a a^x = x$ because $a^{\log_a x} = x$
4. If $\log_a x = \log_a y$, then $x = y$ (One-to-One Property)

Ex: Simplify:

a. $\log_4 1 = 0$

$4^x = 1$

b. $\log_{\sqrt{7}} \sqrt{7} = 1$

c. $6^{\log_6 20} = 20$

d. $\log_3 x = \log_3 (2x - 4)$

$x = 2x - 4$

$4 = x$

Try This: $\log_2 7x = \log_2 (x^2 + 12)$

$7x = x^2 + 12$

$0 = x^2 - 7x + 12$

$0 = (x-4)(x-3)$

$\boxed{x = 4, 3}$

Properties of Natural Logarithms

5. $\ln 1 = 0$ because $e^0 = 1$
6. $\ln e = 1$ because $e^1 = e$
7. $\ln e^x = x$ because $e^{\ln x} = x$
8. If $\ln x = \ln y$, then $x = y$ (One-to-One Property)

Use the properties of natural logarithms to simplify each expression:

a. $\ln\frac{1}{e} = \ln e^{-1} = -1$

b. $e^{\ln 5} = 5$

c. $\frac{\ln 1}{3} = 0$

d. $2\ln e = 2$

Application

The relationship between the number of decibels β and the intensity of a sound I in watts per square meter is

$$\beta = 10\log\left(\frac{I}{10^{-12}}\right)$$

Determine the number of decibels of a sound ~~with an intensity~~ of 2 watts per square meter:

$\beta = 10\log\left(\frac{2}{10^{-12}}\right) \approx \boxed{123\text{ db}}$

Determine the number of decibels of a sound with an intensity of 10^{-2} watt per square meter:

$\beta = 10\log\left(\frac{10^{-2}}{10^{-12}}\right) \approx 10\log_{10}(10^{10}) = \boxed{100}\text{ db}$

Homework: 3.2: p. 236-7 #1, 5, 9, 13, 17, 21, 25, 27-30, 39-44, 45, 49, 53, 57, 61, 65, 79, 85