Pre-Calculus CP 1 – Section 3.3 Notes Name: KEY

Properties of Logarithms

Calculators only have two keys to evaluate logarithms:

- logarithm (LOG) which is base 10
- natural logarithm (LN) which is base "e"

What if you need to evaluate logarithms with other bases?

Change-of-Base Formula

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. The $\log_a x$ can be converted to a different base as follows:

Base b	*Base 10*	*Base e*
$\log_a x = \frac{\log_b x}{\log_b a}$	$\log_a x = \frac{\log x}{\log a}$	$\log_a x = \frac{\ln x}{\ln a}$

Ex 1: Rewrite each logarithm as a ratio of common logarithms and natural logarithms

a) $\log_7 4 = \frac{\log 4}{\log 7} = \frac{\ln 4}{\ln 7}$

b) $\log_2 17 = \frac{\log 17}{\log 2} = \frac{\ln 17}{\ln 2}$

try them in your calculator to ensure you did it right!

We already learned about how to rewrite logs in exponential form, so these inverse properties should make sense

- $\log_b b^x =$ $\underline{x}$
- $b^{\log_b x} =$ $\underline{x}$
- $\log_b b =$ $\underline{1}$
- $\log_b 1 =$ $\underline{0}$

Properties of Logarithms

Ex 2: Use the properties above to evaluate the logarithmic expressions.

a) $3\log_5 5 - \log_5 25 =$ $3(1) - 2 = 1$

b) $\log_4 16^2 + \log_4 8^4 =$ $\log_4 (4^2)^2 + \log_4 (2^3)^4$
$= 4 + \log_4 (2^2)^6 = 4 + \log_4 4^6 = 4 + 6 = 10$

c) $\log_4 2 + \log_4 32 =$ $\frac{1}{2} + \log_4 2^5 = \frac{1}{2} + \log_4 (2^2)^{5/2}$
$= \frac{1}{2} + \log_4 4^{5/2}$
$= 1/2 + 5/2 = 6/2 = 3$

More Properties of Logarithms

Let a be positive real number such that $a \neq 1$ and let n be a real number. If u and v are positive real number, the following properties are true:

1) $\log_a (u \cdot v) = \log_a u + \log_a v$
"the log of a product equals the sum of the logs"

2) $\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v$
"the log of a quotient equals the difference of the logs

3) $\log_a u^n = n \log_a u$
"the log of a number raised to a power equals the power times the log

Ex 3: Expand each logarithmic expression:

a) $\log_4 xy =$ $\boxed{\log_4 x + \log_4 y}$

Properties of Logarithms

b) $\log_3 x^4 = \boxed{4\log_3 x}$

c) $\log_7 xy^3 = \boxed{\log_7 x + 3\log_7 y}$

d) $\log_8 \frac{\sqrt{2x+5}}{7} = \log_8 \sqrt{2x+5} - \log_8 7$

$= \boxed{\frac{1}{2}\log_8(2x+5) - \log_8 7}$

You try: Expand the logarithmic expression:

a) $\log_4 x^3y^2 = \log_4 x^3 + \log_4 y^2$

$= \boxed{3\log_4 x + 2\log_4 y}$

b) $\log_5 \sqrt{\frac{x}{y^5}} = \frac{1}{2}\log_5 \frac{x}{y^5} = \frac{1}{2}(\log_5 x - \log_5 y^5)$

$= \boxed{\frac{1}{2}\log_5 x - \frac{5}{2}\log_5 y}$

c) $\log_5 \sqrt[4]{y} = \boxed{\frac{1}{4}\log_5 y}$

Properties of Logarithms

d) $\log_7\left(\frac{x}{9}\right)=$ $\boxed{\log_7 x - \log_7 9}$

Ex 4: Condense each logarithmic expression.

a) $3\log_3 x+\log_3 y=$ $\log_3 x^3 + \log_3 y$

$\boxed{= \log_3 x^3 y}$

b) $2\log_4 x+\log_4 3-\log_4 y=$ $\log_4 x^2 + \log_4 3 - \log_4 y$

$= \log_4 x^2 \cdot 3 - \log_4 y$

$\boxed{= \log_4 \frac{3x^2}{y}}$

HW: 3.3: p. 243 # 3 - 75 multiples of 3