

Properties of Logarithms

Calculators only have two keys to evaluate logarithms:

- logarithm (LOG) which is base 10
- natural logarithm (LN) which is base “e”

What if you need to evaluate logarithms with other bases?

Change-of-Base Formula

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. The $\log_a x$ can be converted to a different base as follows:

<i>Base b</i>	<i>Base 10</i>	<i>Base e</i>
$\log_a x = \frac{\log_b x}{\log_b a}$	$\log_a x = \frac{\log x}{\log a}$	$\log_a x = \frac{\ln x}{\ln a}$

Ex 1: Rewrite each logarithm as a ratio of common logarithms and natural logarithms

a) $\log_7 4 =$ _____

b) $\log_2 17 =$ _____

try them in your calculator to ensure you did it right!

We already learned about how to rewrite logs in exponential form, so these inverse properties should make sense

- $\log_b b^x =$ _____
- $b^{\log_b x} =$ _____
- $\log_b b =$ _____
- $\log_b 1 =$ _____

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Ex 2: Use the properties above to evaluate the logarithmic expressions.

a) $3\log_5 5 - \log_5 25 =$

b) $\log_4 16^2 + \log_4 8^4 =$

c) $\log_4 2 + \log_4 32 =$

More Properties of Logarithms

Let a be positive real number such that $a \neq 1$ and let n be a real number. If u and v are positive real number, the following properties are true:

1) $\log_a (u \cdot v) = \log_a u + \log_a v$
“the log of a product equals _____”

2) $\log_a \left(\frac{u}{v} \right) = \log_a u - \log_a v$
“the log of a quotient equals _____”

3) $\log_a u^n = n \log_a u$
“the log of a number raised to a power equals _____”

Ex 3: Expand each logarithmic expression:

a) $\log_4 xy =$

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b) $\log_3 x^4 =$

c) $\log_7 xy^3 =$

d) $\log_8 \frac{\sqrt{2x+5}}{7} =$

You try: Expand the logarithmic expression:

a) $\log_4 x^3 y^2 =$

b) $\log_5 \sqrt{\frac{x}{y^5}} =$

c) $\log_5 \sqrt[4]{y} =$

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d) $\log_7\left(\frac{x}{9}\right) =$

Ex 4: Condense each logarithmic expression.

a) $3\log_3 x + \log_3 y =$

b) $2\log_4 x + \log_4 3 - \log_4 y =$