Pre-Calculus CP 1 – Section 3.4 Notes **Name:** KEY

Solving Exponential & Logarithmic Equations

Reminder of Important Properties

- *One-to-One Properties:* $a^x = a^y$ if and only if $x = y$.

  $\log_a x = \log_a y$ if and only if $x = y$.

- *Inverse Properties:* $a^{\log_a x} = x$

  $\log_a a^x = x$

Solving Strategies

1. Rewrite the original equation in a form that allows for the use of the one-to-one properties of exponential or logarithmic functions.
2. Rewrite an exponential equation in logarithmic form, then apply the Inverse Property of logarithmic functions.
3. Rewrite a logarithmic equation in exponential form, then apply the Inverse Property of exponential functions.

Examples

Solve the following exponential equations and approximate to three decimal places (if needed).

1. $4^x = 72$

$\log_4 72 = x$

$x = \frac{\log 72}{\log 4} = 3.085$

2. $3(2^x) = 42$

$2^x = 14$

$\log_2 14 = x = \frac{\log 14}{\log 2} = 3.807$

3. $e^x + 5 = 60$

$e^x = 55$

$\ln 55 = x = 4.007$

4. $2(3^{2t-5}) - 4 = 11$

$3^{2t-5} = \frac{15}{2}$

$\log_3 \frac{15}{2} = 2t - 5$

$t = \frac{\log_3 \frac{15}{2} + 5}{2} = 3.417$

5. $e^{2x}-3e^{x}+2=0$

$(e^x-2)(e^x-1)=0$

$e^x=2$ or $e^x=1$

$\ln 2=x$ or $\ln 1=x$

$\boxed{x=.693 \text{ or } x=0}$

Examples:

Solve the following logarithmic equations and approximate to three decimal places (if needed).

1. $\ln x=2$

$e^2=x$

$x=7.389$

2. $\log_3(5x-1)=\log_3(x+7)$

$5x-1=x+7$

$4x=8$

$x=2$

3. $5+2\ln x=4$

$\ln x^2=-1$

$e^{-1}=x^2$

$x=\sqrt{e^{-1}}=.607$

4. $2\log_5 3x=4$

$\log_5(3x)^2=4$

$5^4=(3x)^2$

$x^2=\frac{625}{9}$

$x=\frac{25}{3}$

Checking for extraneous roots....

5. $\log 5x+\log(x-1)=2$

HW: p. 253-4 #9, 15, 19, 29, 35, 37, 45, 49, 53, 59, 77, 81, 85, 93, 110