EVEN MORE Applications of Exponential and Logarithmic Functions

Half Life

When examining half-life, the important thing to remember is that half-life represents the amount of time for half of the substance to disappear.

You will often use the Law of Uninhibited Growth and Decay when doing half-life problems:

$$A(t)=A_0e^{kt}$$

Examples

1) Iodine 131 is a radioactive material that decays according to the function $A(t)=A_0e^{-0.087t}$ where A_0 is the initial amount present and A is the amount present at time t (in days). What is the half life of Iodine 131?

$$1 = 2e^{-.087t}$$

$$\frac{1}{2} = e^{-.087t}$$

$$\ln\frac{1}{2} = -.087t$$

$$t = \frac{\ln\frac{1}{2}}{-.087} \approx 7.97 \text{ days}$$

2) The half-life of radium is 1690 years. If 10 grams is present now, how much will be present in 50 years?

$$1 = 2e^{k(1690)}$$

$$\ln\frac{1}{2} = 1690k$$

$$k = \frac{\ln\frac{1}{2}}{1690} \approx -0.00041$$

$$A(50) = 10e^{-.00041(50)} \approx 9.8 \text{ gms}$$

Newton's Law of Cooling

The temperature, u, of a heated object at a given time, t, can be modeled by the following function:

$$u(t)=T+(u_0-T)e^{kt},\ k<0$$

where T is the constant temperature of the surrounding medium, u_0 is the initial temperature of the heated object, and k is a negative constant.

Example

An object is heated to 100 degrees Celsius and is then allowed to cool in a room whose air temperature is 30 degrees Celsius.

a) If the temperature of the object is 80°C after 5 minutes, when will the temperature be 50°C?

$80 = 30 + (100-30)e^{k(5)}$

$\frac{5}{7} = e^{5k}$

$5k = \ln 5/7$

$k = \frac{\ln(5/7)}{5} = -.0673$

$50 = 30 + (70)e^{-.0673t}$

$\frac{2}{7} = e^{-.0673t}$

$t = \frac{\ln(\frac{2}{7})}{-.0673} \approx \boxed{18.62 \text{ min}}$

b) Determine the elapsed time before the temperature of the object is 35°C.

$35 = 30 + (70)e^{-.0673t}$

$\frac{1}{14} = e^{-.0673t}$

$t = \frac{\ln(\frac{1}{14})}{-.0673} \approx \boxed{39.22 \text{ min}}$

c) What do ~~e~~ you notice about u(t), the temperature, as time passes?

It gets closer & closer to the room temp. (30°)

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Logistic Growth Model

This is exponential growth, but with inhibitions...in other words, certain factors will *limit* the growth:

$$P(t)=\frac{c}{1+ae^{-bt}},\ c>0,\ b>0$$

where c is the *carrying capacity* because it is the ceiling for how big this value can become.

Example

Fruit flies are placed in a half-pint milk bottle with a banana and yeast plants. Suppose that the fruit fly population after t days is given by $P(t)=\frac{230}{1+56.5e^{-0.37t}}$.

a) What is the carrying capacity of the half-pint bottle?

230 flies

b) How many fruit flies were initially placed in the half-pint bottle?

$$P(0)=\frac{230}{1+56.5e^{-.37(0)}}$$

$$=\frac{230}{1+56.5}=4\text{ flies}$$

c) When will the population of fruit flies be 180?

$$180=\frac{230}{1+56.5e^{-.37t}}$$

$$56.5e^{-.37t}=\frac{23}{18}-\frac{18}{18}=\frac{5}{18}$$

$$e^{-.37t}=\frac{5/18}{56.5}=.004916$$

$$\frac{\ln .004916}{-.37}=t\ \Rightarrow\ \boxed{14.37\text{ days}}$$