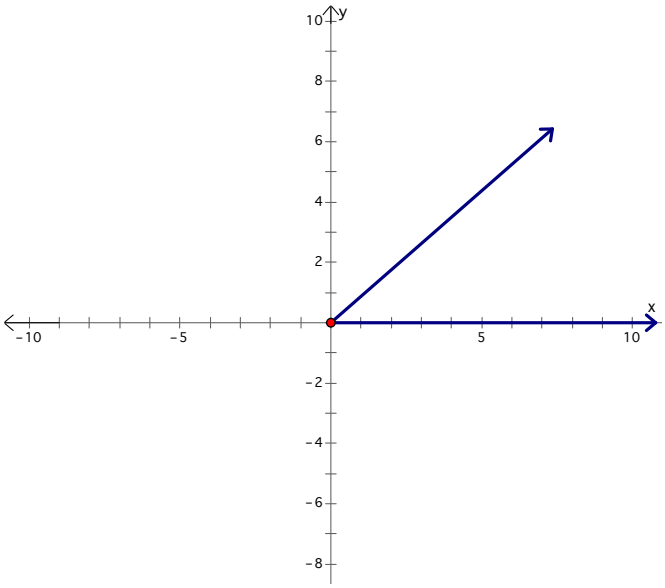
Radian & Degree Measure

Angles

- An angle is determined by rotating a ray about its endpoint.
- The starting position of the ray is the INITIAL side of the angle, and the position after the rotation is the TERMINAL side.
- The endpoint of the ray is the VERTEX of the angle.

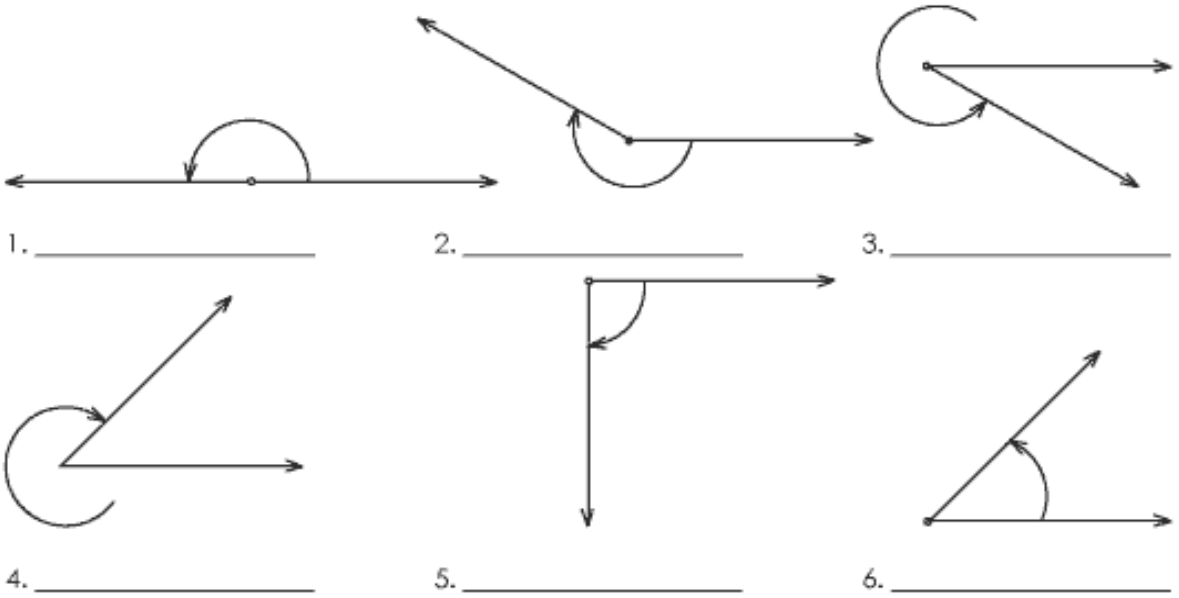
Ex 1 Label the angle with its initial side, terminal side, and vertex.



Standard Position, Positive Angles, Negative Angles

- In the coordinate plane, when the vertex of an angle is at the origin, and the initial side is on the positive x-axis, the angle is said to be in STANDARD POSITION.
- Positive angles are formed by a COUNTER- clockwise rotation, and negative angles are formed by a clockwise rotation.
- Angles are typically labeled by Greek letters sch as α (alpha) β (beta) or θ (theta) as well as uppercase letters such as A, B, or C.

Ex 2 Estimate the degree measure of each of the following angles.

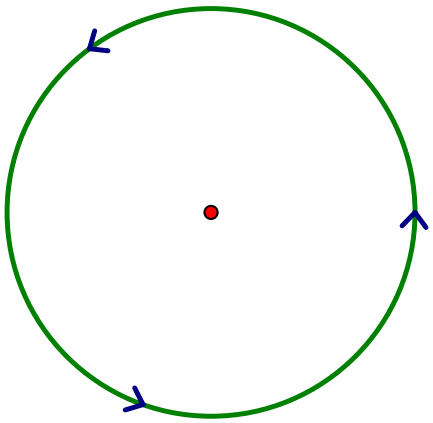


Ex 3 What quadrant do the following angles' terminal side lie in?

A) 130° B) 350° C) −100° D) 200°

Radian Measure

The measure of an angle is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in **radians.** This type of measure is especially useful in calculus and is helpful in connecting linear units with angle measure.



Remember: One full revolution (360°) has measure _____ radians.

Radians compared to the degree measure

The first common way you learned to measure angles is in **degrees**, where a full (**counterclockwise**) revolution corresponds to 360°, a half-revolution to 180°, a quarter-revolution to 90°, and so on.

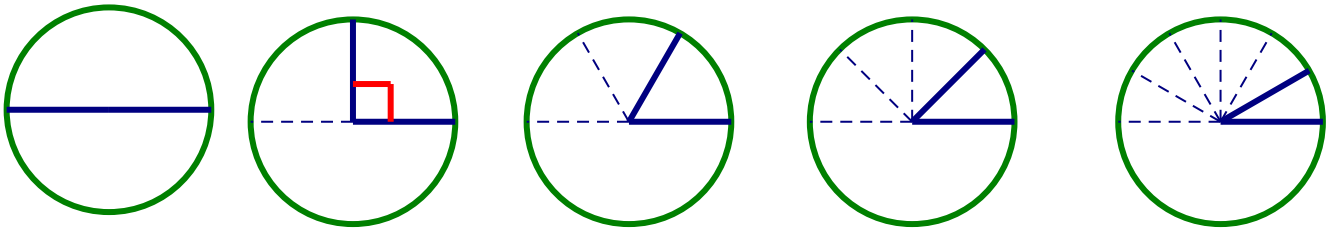
Ex 4 Given that <u>one full revolution measures 2π radians</u>,

$\frac{1}{2}$ revolution = _____ radians = _____ degrees

$\frac{1}{4}$ revolution = _____ radians = _____ degrees

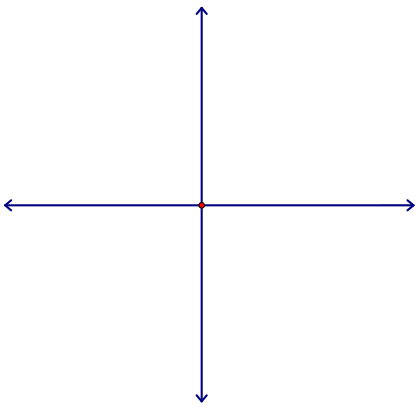
$\frac{1}{6}$ revolution = _____ radians = _____ degrees

Ex 5 How many radians are shown in each diagram?



_____ _____ _____ _____ _____

Ex 6 Label each quadrantal angle in radian measure



Given that 2π radians corresponds to one complete revolution, degrees and radians are related by the equations:

$360° = 2\pi$ radians and $180° = \pi$ radians

Therefore, 1° = _____ radians and 1 radian = _____ degrees.

Converting Degrees ⇔ Radians

- To convert from Degrees to Radians, multiply degrees by _____.
- To convert from Radians to Degrees, multiply radians by _____.

Ex 7 Most frequently used conversions:

30° = _____ radians 45° = _____ radians 60° = _____ radians

90° = _____ radians 180° = _____ radians

270° = _____ radians 360° = _____ radians

Ex 8 Convert from Degrees → Radians

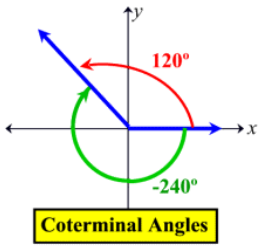
a. 135° b. –210°

Ex 89 Convert from Radians → Degrees

a. $-\frac{5\pi}{6}$ radians b. $\frac{2\pi}{3}$ radians

Coterminal Angles

When two or more angles have the same initial and terminal sides, they are "coterminal"



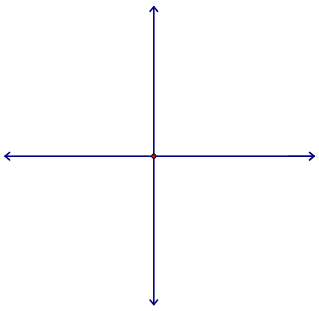
In radians, you can find an angle that is coterminal to a given angle θ by **adding or subtracting** 2π (1 full revolution) or any **multiple of** 2π

Ex 4 Find 3 angles coterminal to $110°$

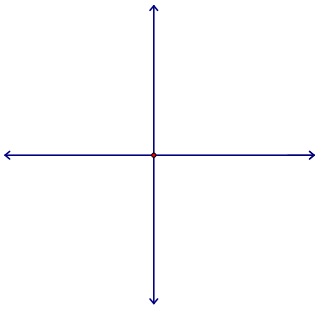
How many angles are coterminal to $110°$? _____

Ex 5 Finding Coterminal Angles in radian measure

a. For the positive angle $\frac{13\pi}{6}$, find a positive coterminal angle, and sketch.



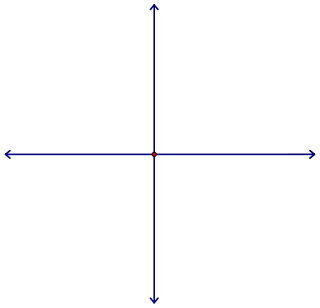
b. For the positive angle $\frac{3\pi}{4}$, find a negative coterminal angle, and sketch.



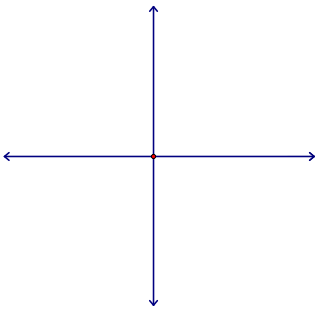
Section 4.1 HW Day 1 : p.290-2 #7, 13, 17, 19, 25-37 odd, 51-69 odd

Additional Practice

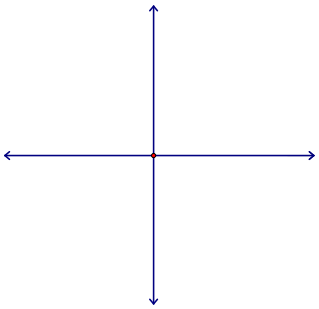
1) Find a positive coterminal angle for $\theta = 100°$. Sketch both angles in standard position.



2) Sketch $-120°$ in standard position. Find two more negative angles that are coterminal with $-120°$.

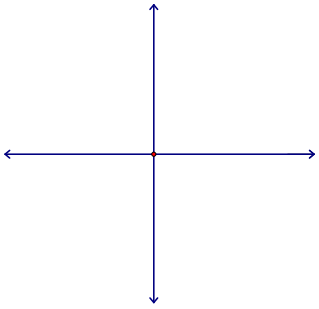


3) For the negative angle $-\frac{2\pi}{3}$, find a positive coterminal angle, and sketch.



4) Determine the quadrant in which $\frac{9\pi}{8}$ lies.

5) Sketch $\frac{5\pi}{2}$ in standard position.



6) Convert from degrees to radians: $150°$

7) Convert from radians to degrees: $\frac{5\pi}{4}$

More from Lesson 4.1 with Applications!

Degrees, Minutes, Seconds vs Decimal Degrees

Minute = 1' = _____ Second = 1" = _____

Ex 1 Convert from DMS to Decimal Degrees: 74° 8' 14"

Ex 2 Convert from Decimal Degrees to DMS: 34.817°

Ex 3 You try:

a. 34° 51'35" → decimal degrees b. 102.3771 → DMS

Arc Length

For a circle of radius r, a central angle of θ intercepts an **arc of length s** given by

$$s = r\theta$$ where θ is measured in RADIANS

Ex 6 A circle has a radius of 5 cm.
Find the length of the arc intercepted by a central angle of 150°.

Linear and Angular Speeds

Consider a particle moving at constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed v** of the particle is

$$v = \frac{s}{t} = \frac{arc\ length}{time}$$

Also, if θ is the angle (again, in radian measure) corresponding to the arc length s, then the **angular speed ω** of the particle is

$$\omega = \frac{\theta}{t} = \frac{central\ angle}{time}$$

Ex 7: Ms. Chen and her newphew, Brady, went on a Ferris wheel for the first time this summer! It was designed for 4 year olds, so it had an 80 foot diameter and made about 1.25 revolutions per minute. Find the angular and linear speeds of the wheel so Ms. Chen can tell Brady how FAST they went!

Area of a Sector of a Circle

For a circle of radius r, the area A of a sector of the circle with central angle of θ is given by

$$A = \frac{1}{2} r^2 \theta$$, where θ is measured in RADIANS

Ex 8: A pizza is about to be split up and you want to be sure you get a lot of it. The diameter is 16 inches and you want a piece with a central angle of 170°. Find the area of the piece you plan to get.

Section 4.1 HW Day 2 : p.290 #39, 41, 71-77 odd, 83, 87, 91, 103, 107

Additional Practice

1) Convert from DMS to Decimal Degrees: 57°32'45"

2) Convert from Decimal Degrees to DMS: 97.125°

3) A carousel with a 50 foot diameter makes 4 revolutions per minute. Find the angular speed of the carousel in radians per minute, and the linear speed of the platform rim.