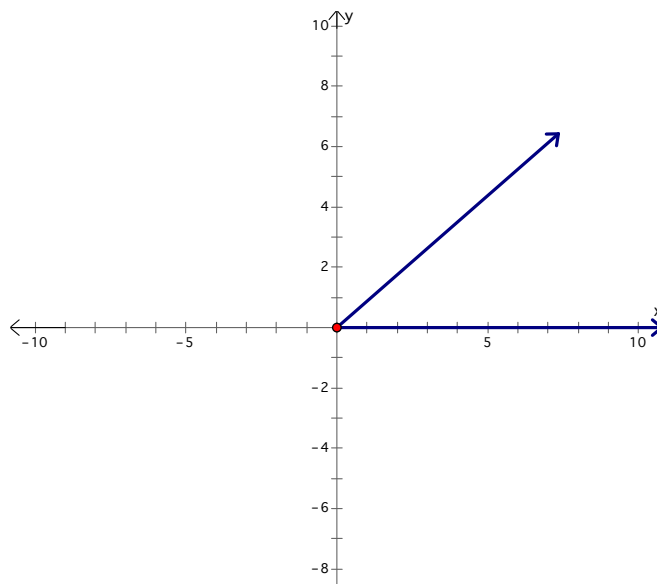


Angles

- An angle is determined by rotating a ray about its endpoint.
- The starting position of the ray is the INITIAL side of the angle, and the position after the rotation is the TERMINAL side.
- The endpoint of the ray is the VERTEX of the angle.

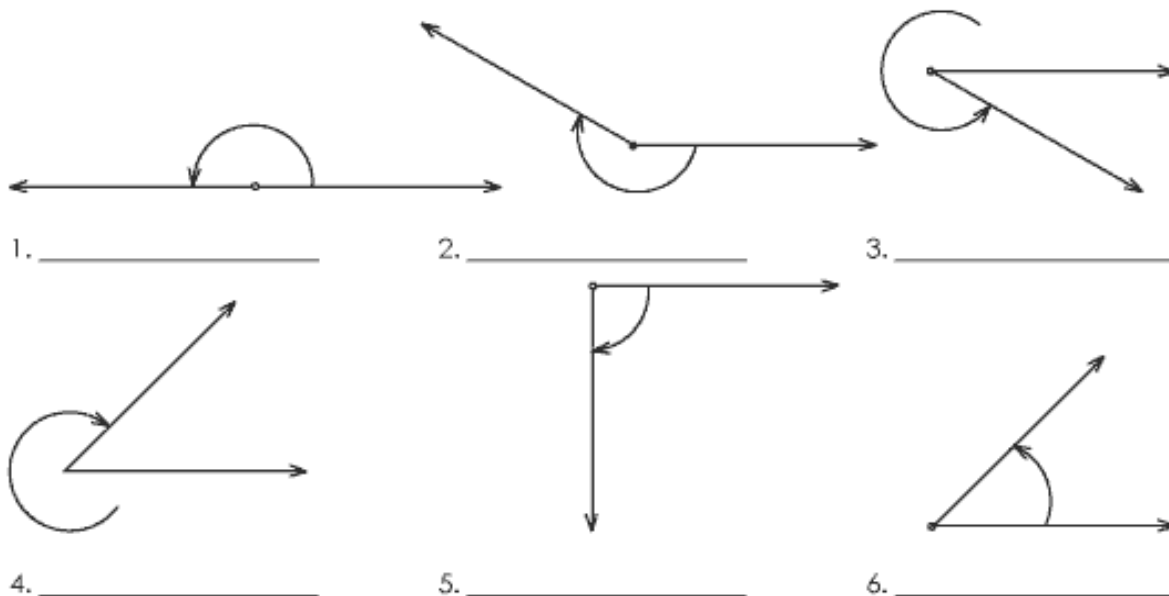
Ex 1 Label the angle with its initial side, terminal side, and vertex.



Standard Position, Positive Angles, Negative Angles

- In the coordinate plane, when the vertex of an angle is at the origin, and the initial side is on the positive x-axis, the angle is said to be in STANDARD POSITION.
- Positive angles are formed by a COUNTER- clockwise rotation, and negative angles are formed by a clockwise rotation.
- Angles are typically labeled by Greek letters such as α (alpha) β (beta) or θ (theta) as well as uppercase letters such as A, B, or C.

Ex 2 Estimate the degree measure of each of the following angles.

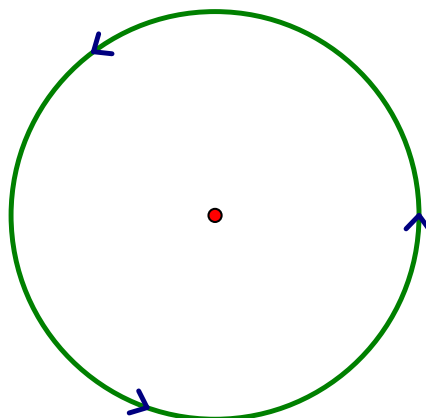


Ex 3 What quadrant do the following angles' terminal side lie in?

- A) 130° B) 350° C) -100° D) 200°

Radian Measure

The measure of an angle is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in **radians**. This type of measure is especially useful in calculus and is helpful in connecting linear units with angle measure.



Remember: One full revolution (360°) has measure _____ radians.

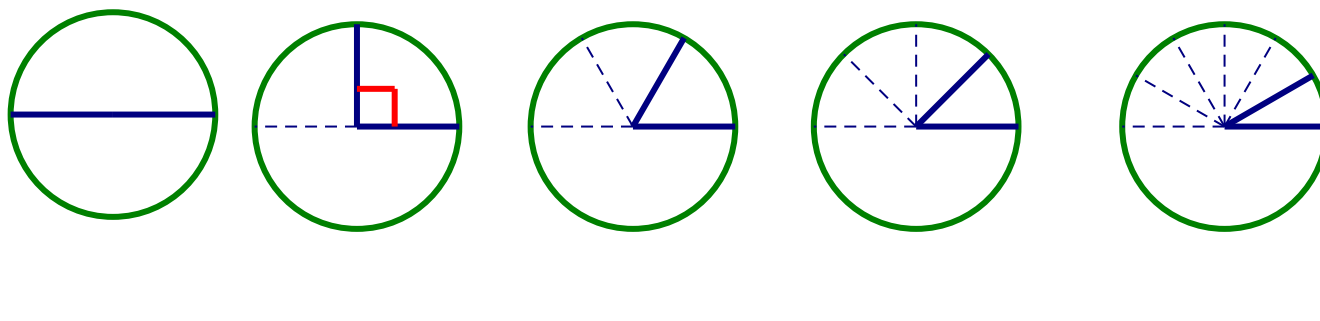
Ex 4 Given that one full revolution measures 2π radians,

$\frac{1}{2}$ revolution = _____ radians = _____ degrees

$\frac{1}{4}$ revolution = _____ radians = _____ degrees

$\frac{1}{6}$ revolution = _____ radians = _____ degrees

Ex 5 How many radians are shown in each diagram?



Radians compared to the degree measure

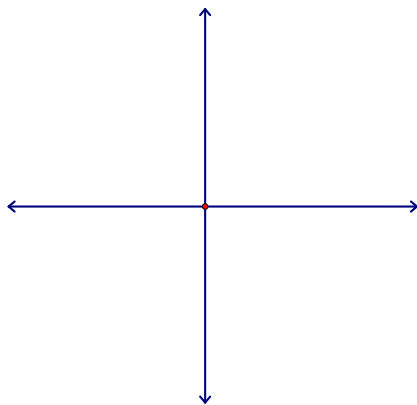
The first common way you learned to measure angles is in **degrees**, where a full (**counterclockwise**) revolution corresponds to 360° , a half-revolution to 180° , a quarter-revolution to 90° , and so on.

Given that 2π radians corresponds to one complete revolution, degrees and radians are related by the equations:

$360^\circ = 2\pi$ radians and $180^\circ = \pi$ radians

Therefore, $1^\circ = \underline{\hspace{1cm}}$ radians and 1 radian = $\underline{\hspace{1cm}}$ degrees.

Ex 6 Label each quadrantal angle in radian measure



Converting Degrees \leftrightarrow Radians

- To convert from Degrees to Radians, multiply degrees by _____.
- To convert from Radians to Degrees, multiply radians by _____.

Ex 7 Convert from Degrees \rightarrow Radians

a. 135°

b. -210°

Ex 8 Convert from Radians \rightarrow Degrees

a. $-\frac{5\pi}{6}$ radians

b. $\frac{2\pi}{3}$ radians

Ex 9 Actually, these angles should be ones that you know VERY quickly since they will be used a lot

$30^\circ =$ _____ radians

$45^\circ =$ _____ radians

$60^\circ =$ _____ radians

$90^\circ =$ _____ radians

$180^\circ =$ _____ radians

$270^\circ =$ _____ radians

$360^\circ =$ _____ radians

Section 4.1 HW Day 1 : p.290-2 #7, 13, 17, 19, 25-37 odd, 51-69odd

Degrees, Minutes, Seconds vs Decimal Degrees

Minute = 1' = _____

Second = 1" = _____

Ex 1 Convert from DMS to Decimal Degrees: $74^\circ 8' 14''$

Ex 2 Convert from Decimal Degrees to DMS: 34.817°

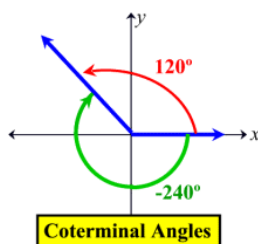
Ex 3 You try:

a. $34^\circ 51' 35'' \rightarrow$ decimal degrees

b. $102.3771 \rightarrow$ DMS

Coterminal Angles

When two or more angles have the same initial and terminal sides, they are “coterminal”



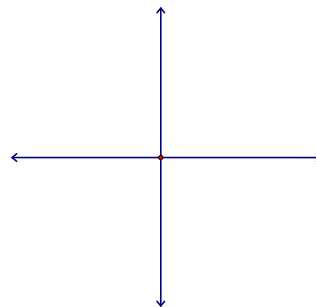
In radians, you can find an angle that is coterminal to a given angle θ by **adding or subtracting** 2π (1 full revolution) or any **multiple of** 2π

Ex 4 Find 3 angles coterminal to 110°

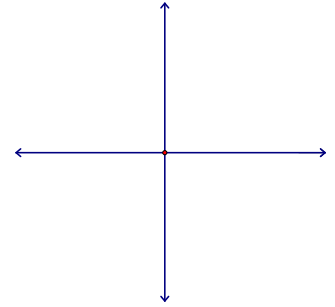
How many angles are coterminal to 110° ? _____

Ex 5 Finding Coterminal Angles in radian measure

- a. For the positive angle $\frac{13\pi}{6}$, find a positive coterminal angle, and sketch.



- b. For the positive angle $\frac{3\pi}{4}$, find a negative coterminal angle, and sketch.



Arc Length

For a circle of radius r , a central angle of θ intercepts an **arc of length s** given by

$$s = r\theta \quad \text{where } \theta \text{ is measured in RADIANS}$$

- Ex 6** A circle has a radius of 5 cm.
Find the length of the arc intercepted by a central angle of 150° .

Linear and Angular Speeds

Consider a particle moving at constant speed along a circular arc of radius r . If s is the length of the arc traveled in time t , then the **linear speed** v of the particle is

$$v = \frac{s}{t} = \frac{\text{arc length}}{\text{time}}$$

Also, if θ is the angle (again, in radian measure) corresponding to the arc length s , then the **angular speed** ω of the particle is

$$\omega = \frac{\theta}{t} = \frac{\text{central angle}}{\text{time}}$$

Ex 7: Ms. Collins and her son, Brady, went on a Ferris wheel for the first time this summer! It was designed for 4 year olds, so it had an 80 foot diameter and made about 1.25 revolutions per minute. Find the angular and linear speeds of the wheel so Ms. Collins can tell Brady how FAST they went!

Area of a Sector of a Circle

For a circle of radius r , the area A of a sector of the circle with central angle of θ is given by

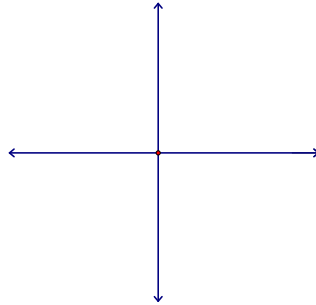
$$A = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is measured in RADIANS}$$

Ex 8: A pizza is about to be split up and you want to be sure you get a lot of it. The diameter is 16 inches and you want a piece with a central angle of 170° . Find the area of the piece you plan to get.

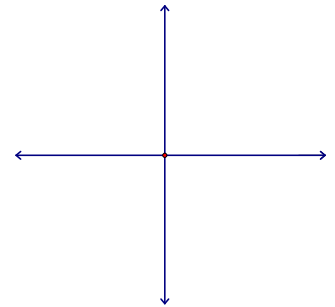
Section 4.1 HW Day 2 : p.290 #39, 41, 71-77 odd, 83, 87, 91, 103, 107

Additional Practice

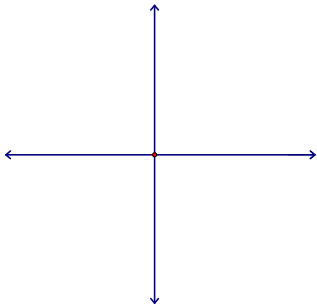
- 1) Find a positive coterminal angle for $\theta = 100^\circ$. Sketch both angles in standard position.



- 2) Sketch -120° in standard position. Find two more negative angles that are coterminal with -120° .



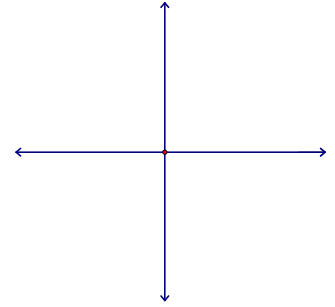
- 3) For the negative angle $-\frac{2\pi}{3}$, find a positive coterminal angle, and sketch.



- 4) Determine the quadrant in which $\frac{9\pi}{8}$ lies.

PreCalculus Notes – Section 4.1 Radian and Degree Measure

5) Sketch $\frac{5\pi}{2}$ in standard position.



6) Find two coterminal angles for $\theta = \frac{2\pi}{3}$, one positive and one negative.

7) Convert from degrees to radians: 150°

8) Convert from radians to degrees: $\frac{5\pi}{4}$

9) Convert from DMS to Decimal Degrees: $57^\circ 32' 45''$

10) Convert from Decimal Degrees to DMS: 97.125°

11) A carousel with a 50 foot diameter makes 4 revolutions per minute. Find the angular speed of the carousel in radians per minute, and the linear speed of the platform rim.