Right Triangle Trigonometry

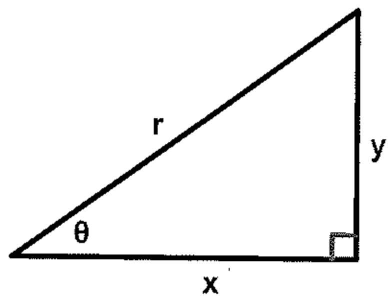
Before we can continue our exploration of radians in the coordinate plane, we need to be sure you remember how to apply trig ratios to right triangles.

Given the triangle to the right, define the following trig ratios:

sine => $\sin\theta = \frac{y}{r}$

cosine => $\cos\theta = \frac{x}{r}$

tangent => $\tan\theta = \frac{y}{x}$



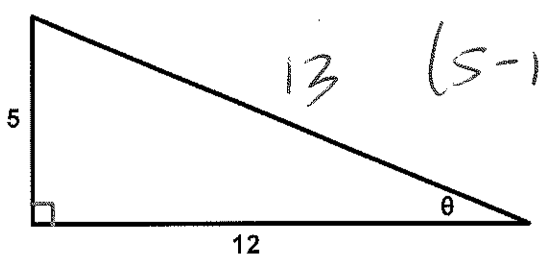
The next three trig ratios may be unfamiliar- they are reciprocals of the ones above;

cosecant => $\csc\theta = \frac{r}{y}$

secant => $\sec\theta = \frac{r}{x}$

cotangent => $\cot\theta = \frac{x}{y}$

Ex. 1) Find the values of the six trig functions of θ:



13 (5-12-13 Δ)

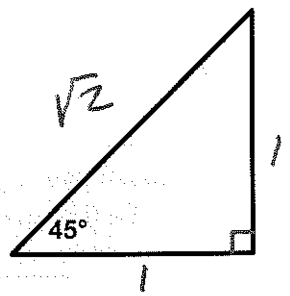
$\sin\theta = 5/13$ $\cos\theta = 12/13$ $\tan\theta = 5/12$

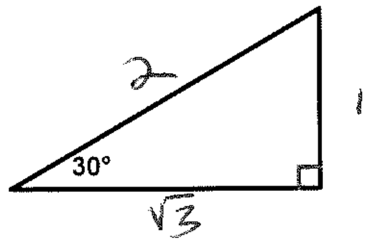
$\csc\theta = 13/5$ $\sec\theta = 13/12$ $\cot\theta = 12/5$

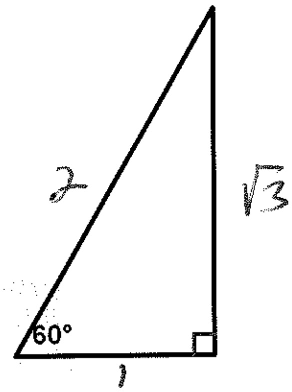
In certain triangles (SPECIAL ones) the ratios among the sides are in a consistent pattern, so if you are given just ONE side, you can figure out both the others.

Special Right Triangles:

45°-45°-90° and 30°-60°-90°







If $\theta = 45°$, then	If $\theta = 30°$, then	If $\theta = 60°$, then
$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\sin 30° = \frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$
$\tan 45° = 1$	$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\tan 60° = \sqrt{3}$
$\csc 45° = \sqrt{2}$	$\csc 30° = 2$	$\csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\sec 45° = \sqrt{2}$	$\sec 30° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	$\sec 60° = 2$
$\cot 45° = 1$	$\cot 30° = \sqrt{3}$	$\cot 60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

These triangles have ratios you need to MEMORIZE! **I am not joking**. If you remember your special right triangles, you will have a much easier time remembering the trig ratios. I mean it- ***memorize your special right triangles!!***

$\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{y}{r}$

$\Rightarrow y = r\sin\theta$

$\cos\theta = \frac{x}{r}$

$\Rightarrow x = r\cos\theta$

$\therefore \tan = \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}$

$= \frac{\sin\theta}{\cos\theta}$

Fundamental Trig Identities

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta} \quad \cos\theta = \frac{1}{\sec\theta} \quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identity:

$\sin^2\theta + \cos^2\theta = 1$

more Pythagorean identities:

$1 + \cot^2\theta = \csc^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

now let's derive the other two…

$\frac{\sin^2\theta + \cos^2\theta = 1}{\sin^2}$

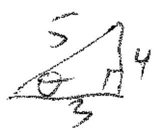
$1 + \cot^2\theta = \csc^2\theta$

$\frac{\sin^2\theta + \cos^2\theta = 1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

Ex. 2) Let θ be an acute angle such that $\sin\theta = 0.8$. Without using a calculator, find the values of $\cos\theta$ and $\tan\theta$.

$\sin\theta = .8 = \frac{4}{5}$



$\therefore \cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

Ex. 3) Let θ be an acute angle such that $\tan\theta = 3$. Without using a calculator, find the values of $\cot\theta$ and $\sec\theta$.

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{3}$

$\sec^2\theta = \tan^2\theta + 1$

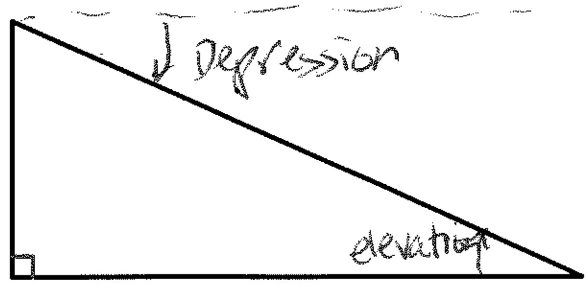
$= 3^2 + 1 = 10$

$\sec\theta = \sqrt{10}$

HW (day 1) 4.3: p. 308 #3, 6, 10, 17-22, 27, 28, 33-35, 59, 60

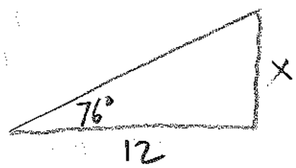
Applications of Right Triangle Trig

Angle of Elevation & Angle of Depression:



Draw a diagram and LABEL appropriate parts before solving:

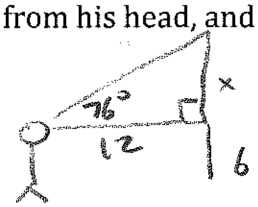
Ex. 4) Peter stands 12 ft from the base of a tree. The <u>angle of elevation</u> from his feet to the top of the tree is 76°. How tall is the tree?



$\tan 76° = \frac{x}{12}$

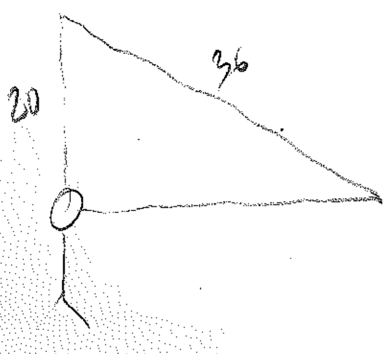
$x = 12(\tan 76°) \approx 48.13$ ft

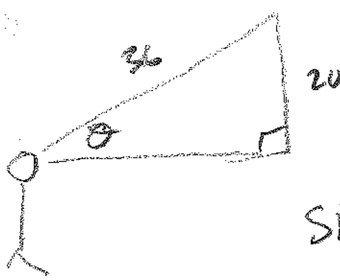
How would you do this problem differently if I told you that the angle of elevation was from his head, and he is 6 feet tall?



Add 6 to the height!

Ex. 5) A kite has a string with length 36 feet is being flown at a height of 20 feet above Mary Kate's head. What is the angle at which Mary Kate needs to tilt her head to look at the kite?





$\sin\theta = \frac{20}{36}$

$\Rightarrow \theta \approx 33.75$ ft

HW 4.3 day 2: complete the packet of word problems
Also, suggested from the book: p. 309-310 #63, 65-68