## 4.7 – Inverse Trig Functions (Day 2)

## Compositions of Functions

Recall: If $f(x)$ and $f^{-1}(x)$ are truly inverse functions, then for all x values in the domains of $f(x)$ and $f^{-1}(x)$, the following is true:

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = 1$

**Inverse Properties of Trigonometric Functions:**

Always true

$$\sin\left(\sin^{-1}(x)\right) = x$$
$$\cos\left(\cos^{-1}(x)\right) = x$$
$$\tan\left(\tan^{-1}(x)\right) = x$$

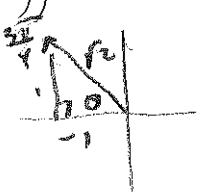
Only true for x-values in the "restricted" domains

$$\sin^{-1}\left(\sin(x)\right) = x$$
$$\cos^{-1}\left(\cos(x)\right) = x$$
$$\tan^{-1}\left(\tan(x)\right) = x$$

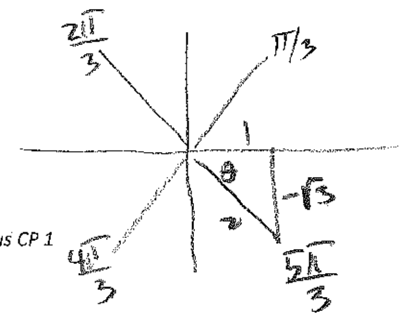
**Example 1**: Find the exact value of the following:

a) $\sin(\sin^{-1}(1)) = 1$

b) $\cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right) = \frac{3\pi}{4}$




c) $\sin^{-1}\left(\underbrace{\sin\left(\frac{5\pi}{3}\right)}_{}\right) = -\frac{\pi}{3}$ ($-\sqrt{3}/2$)

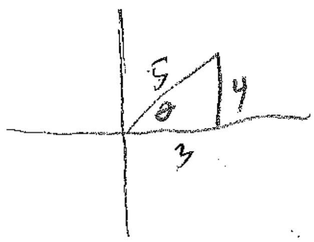



d) $\tan\left(\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{\sqrt{3}}{2}$

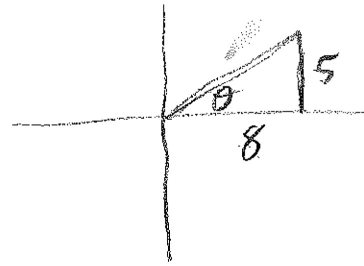
# 4.7 – Inverse Trig Functions (Day 2)

**Example 2**: Find the exact value of the following (hint: draw a triangle!):

a) $\sec\left(\overset{\theta}{\arcsin}\left(\frac{4}{5}\right)\right) = \frac{5}{3}$

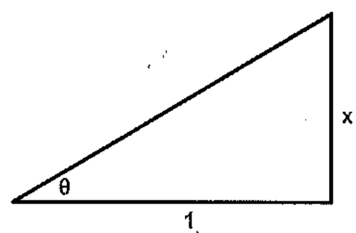



b) $\cot\left(\arctan\left(\frac{5}{8}\right)\right) = \frac{8}{5}$




**Exploration**

Use the triangle at the right to answer the following:




1) Find $\tan\theta$ $= \frac{x}{1} = x$

2) Find $\tan^{-1}(x)$ $= \theta$

3) Find the hypotenuse of the triangle as a function of x

$$x^2 + 1^2 = c^2$$
$$\Rightarrow c = \sqrt{x^2+1}$$

4) Find $\sin\left(\tan^{-1}(x)\right)$ as a ratio involving no trig functions

$$= \sin(\theta) = \frac{x}{\sqrt{x^2+1}} = \frac{x\sqrt{x^2+1}}{x^2+1}$$

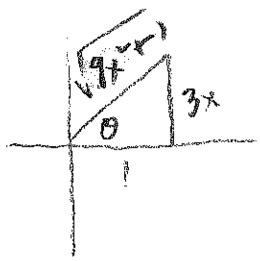
5) Find $\sec\left(\tan^{-1}(x)\right)$ as a ratio involving no trig functions

$$\sec\theta = \frac{\sqrt{x^2+1}}{1} = \sqrt{x^2+1}$$

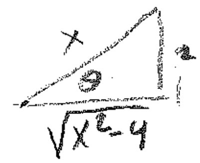
## 4.7 – Inverse Trig Functions (Day 2)

**Example 3:** Write an algebraic expression that is equivalent to the given expression (hint: draw a triangle!)

a) $\sec(\arctan(3x))$ $= \sec\theta = \sqrt{9x^2+1}$

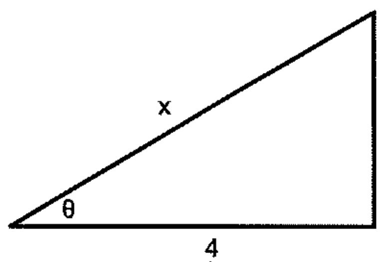



b) $\cot\left(\arcsin\left(\frac{2}{x}\right)\right)$ $= \cot(\theta) = \frac{\sqrt{x^2-4}}{2}$




**Example 4:** Use an inverse trigonometric function to write θ as a function of x.

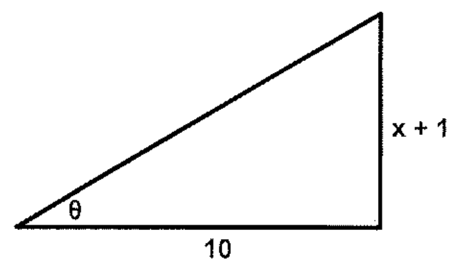
a)




$\cos\theta = \frac{4}{x}$

$\Rightarrow \arccos\left(\frac{4}{x}\right) = \theta$

b)




$\tan\theta = \frac{x+1}{10}$

$\Rightarrow \theta = \arctan\left(\frac{x+1}{10}\right)$

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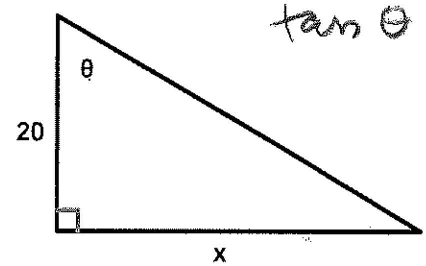
**Example 5:** Application

A security car with its spotlight on is parked 20 meters from a warehouse. Consider θ and x as shown in the figure.

a. Write θ as a function of x

$\theta = \arctan\left(\frac{x}{20}\right)$

$\tan\theta = \frac{x}{20}$




b. Find θ when x = 5 meters.

$\theta = \arctan\left(\frac{5}{20}\right)$
$= \arctan\left(\frac{1}{4}\right) \approx .24°$

c. Find θ when x = 12 meters.

$\theta = \arctan\left(\frac{12}{20}\right) = \arctan\left(\frac{3}{5}\right)$
$\approx .54°$

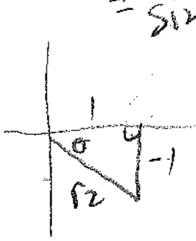

**Example 5:** More Practice!

a) $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$

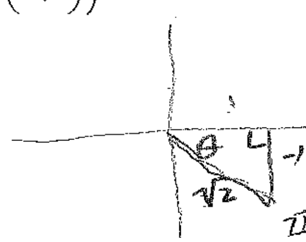

b) $\sin^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$


c) $\sin(\tan^{-1}(-1))$

$= \sin\left(\frac{-\pi}{4}\right) = -\frac{\sqrt{2}}{2}$




d) $\cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$




$\frac{7\pi}{4} = -\frac{\pi}{4}$

**Homework: 4.7 Exercises Day 2: p. 349# 37, 39, 41, 49 - 67 odd, 91, 92**