

Section 5.3 – Solving Trig Equations

To solve trigonometric equations we use standard algebraic techniques such as collecting like terms and factoring. ALWAYS try to isolate the trig function in the equation.

Examples:

Solve the following trig equations and give all possible solutions in the interval $[0, 2\pi)$.

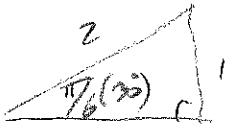
1. Verifying Solutions of Trig Equations

Decide if the following solve this equation: $\sin 2x - \frac{1}{2} = 0$

a) Is $x = \frac{\pi}{12}$ a solution?

$$\sin 2 \frac{\pi}{12} = \frac{1}{2} ?$$

$$\sin \frac{\pi}{6} = \frac{1}{2} \checkmark$$

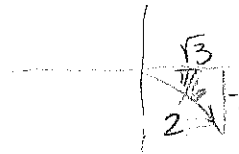


b) Is $x = \frac{11\pi}{12}$ a solution?

$$\sin \frac{22\pi}{12} = \frac{1}{2} ?$$

$$\sin \frac{11\pi}{6} = \frac{1}{2} ?$$

NO



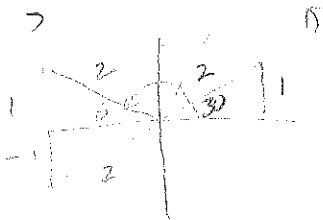
$$\sin \frac{11\pi}{6} = -\frac{1}{2} !$$

2. Solving by getting the Trig Function alone

$$2\sin\theta = 1$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$



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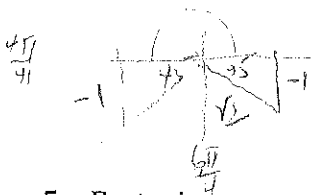
3. Collecting Like Terms

$$\sin\theta + \sqrt{2} = -\sin\theta$$

$$2\sin\theta = -\sqrt{2}$$

$$\sin\theta = -\frac{\sqrt{2}}{2}$$

$$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$$



5. Factoring

$$\cot\theta \cos^2\theta = \cot\theta$$

$$\cot\theta \cos^2\theta - \cot\theta = 0$$

$$\cot\theta(\cos^2\theta - 1) = 0$$

$$\cot\theta = 0$$

$$\Rightarrow \frac{\cos\theta}{\sin\theta} = 0$$

$$\Rightarrow \cos\theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos^2\theta - 1 = 0$$

$$\cos^2\theta = 1$$

$$\cos\theta = \pm 1$$

$$\theta = 0, \pi$$



7. Rewriting with a Single Trig Function

$$2\sin^2\theta + 3\cos\theta - 3 = 0$$

$$2(1 - \cos^2\theta) + 3\cos\theta - 3 = 0$$

$$2 - 2\cos^2\theta + 3\cos\theta - 3 = 0$$

$$-2\cos^2\theta + 3\cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta - 1) = 0$$

$$2\cos\theta - 1 = 0$$

$$\cos\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\theta = 0$$



4. Extracting Square Roots

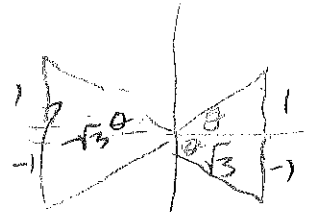
$$3\tan^2\theta - 1 = 0$$

$$3\tan^2\theta = 1$$

$$\tan^2\theta = \frac{1}{3}$$

$$\tan\theta = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$



6. Factoring and Equation of Quadratic Type

$$2\sin^2\theta - \sin\theta - 1 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$$2\sin\theta + 1 = 0$$

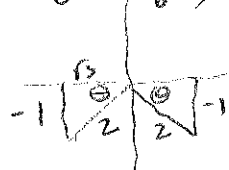
$$\sin\theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\sin\theta - 1 = 0$$

$$\sin\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{2}$$



8. Squaring and Converting to Quadratic Type

(check for extraneous roots)

$$\cos\theta + 1 = \sin\theta$$

$$(\cos\theta + 1)^2 = \sin^2\theta$$

$$\cos^2\theta + 2\cos\theta + 1 = 1 - \cos^2\theta$$

$$2\cos^2\theta + 2\cos\theta = 0$$

$$2\cos\theta(\cos\theta + 1) = 0$$

$$2\cos\theta = 0$$

$$\cos\theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos\theta + 1 = 0$$

$$\cos\theta = -1$$

$$\Rightarrow \theta = \pi$$

HW: 5.3, p.396 #2, 5, 7, 10, 11, 14, 15, 22, 23, 26, 27, 30