Section 5.4 – Sum and Difference Formulas

Don't worry, you do not have to memorize the following formulas, ***but*** you have to know how to use them

$$\sin(\theta+\beta)=\sin\theta\cos\beta+\cos\theta\sin\beta$$

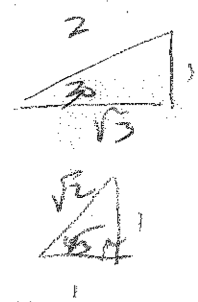
$$\sin(\theta-\beta)=\sin\theta\cos\beta-\cos\theta\sin\beta$$

$$\cos(\theta+\beta)=\cos\theta\cos\beta-\sin\theta\sin\beta$$

$$\cos(\theta-\beta)=\cos\theta\cos\beta+\sin\theta\sin\beta$$

$$\tan(\theta+\beta)=\frac{\tan\theta+\tan\beta}{1-\tan\theta\tan\beta}$$

$$\tan(\theta-\beta)=\frac{\tan\theta-\tan\beta}{1+\tan\theta\tan\beta}$$



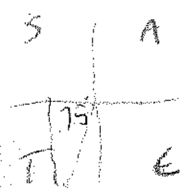
The goal here is to find two angles with REFERENCE angles of 30°, 45° or 60° that can be combined to get the angle you need.

Ex. 1) Find the EXACT value of the following – no decimals!

a. $\sin 75°$ $= \sin(30+45) = \sin 30\cos 45 - \cos 30 \sin 45$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$



b. $\cos 255° = \cos(75°) = -\cos(30+45) = -(\cos 30\cos 45 - \sin 30 \sin 45)$

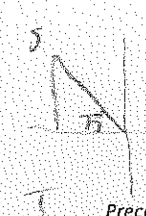
$= -\left(\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\right) = -\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

210+45

$= -\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$

c. $\tan 105°$ $= -\tan(75) = -\tan(30+45)$

60+45



$= -\frac{\tan 30 + \tan 45}{1-\tan 30\tan 45} = -\left(\frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}}\right)$

$= -\left(\frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}\right) = -\left(\frac{1+\sqrt{3}}{\sqrt{3}-1}\right) = \frac{1+\sqrt{3}}{1-\sqrt{3}}\left(\frac{1+\sqrt{3}}{1+\sqrt{3}}\right)$

$\frac{1+2\sqrt{3}+3}{1-3} = \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}$

Section 5.4 – Sum and Difference Formulas

Ex. 2) Write the expression as the sine, cosine, or tangent of the angle and find the exact value: (so basically, match to one of the formulas on the first page)

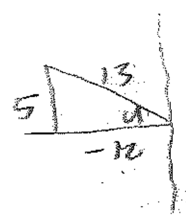
a. $\sin 42^\circ \cos 12^\circ - \cos 42^\circ \sin 12^\circ$ $= \sin(42-12) = \sin 30^\circ = \frac{1}{2}$

b. $\cos 27^\circ \cos 18^\circ - \sin 27^\circ \sin 18^\circ$ $= \cos(27+18) = \cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

c. $\dfrac{\tan 75^\circ - \tan 15^\circ}{1+\tan 75^\circ \tan 15^\circ}$ $= \tan(75-15) = \tan 60 = \frac{\sqrt{3}}{1} = \sqrt{3}$

Ex. 3) Find the exact value (no decimals) of the trig function given that

$\sin u = \dfrac{5}{13}$ and $\cos v = \dfrac{-3}{5}$ (**both** u and v are in Quadrant II)

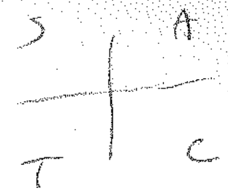


$\sin(v-u) = \sin v \cos u - \cos v \sin u$

$= \frac{4}{5} \cdot \frac{-12}{13} - \frac{-3}{5} \cdot \frac{5}{13}$

$= -\frac{48}{65} + \frac{15}{65} = -\frac{33}{65}$

Section 5.4 – Sum and Difference Formulas



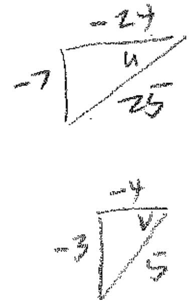
Ex. 4) Find the exact value of the trig function given that $\sin u = \frac{-7}{25}$ and $\cos v = \frac{-4}{5}$.

If both u and v are in the same quadrant, they must be in Quadrant 3

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$= \frac{\frac{7}{24} - \frac{3}{4}}{1 + \frac{7}{24}\left(\frac{3}{4}\right)} = \frac{\frac{7-18}{24}}{\frac{96+21}{96}}$$

$$= \frac{-11}{24} \cdot \frac{96^{4}}{117} = \frac{-44}{117}$$



Ex. 5) Verify the Identity (Yes, it's back!)

$$\cos(x+y) + \cos(x-y) = 2\cos x \cos y$$

$$\cos(x+y) + \cos(x-y) = (\cos x \cos y - \sin x \sin y) + (\cos x \cos y + \sin x \sin y)$$

$$= 2\cos x \cos y \checkmark$$

HW: p. 404-405 #1, ~~7~~, 9, 15-~~18~~, 23-~~26~~, 31, ~~32~~, 37, ~~39~~, ~~41~~, ~~56~~, 63

15, 17 23, 25

Section 5.4 – Review of Sum and Difference Formulas

$$\sin(\theta+\beta)=\sin\theta\cos\beta+\cos\theta\sin\beta$$

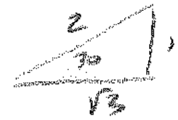
$$\sin(\theta-\beta)=\sin\theta\cos\beta-\cos\theta\sin\beta$$

$$\cos(\theta+\beta)=\cos\theta\cos\beta-\sin\theta\sin\beta$$

$$\cos(\theta-\beta)=\cos\theta\cos\beta+\sin\theta\sin\beta$$

$$\tan(\theta+\beta)=\frac{\tan\theta+\tan\beta}{1-\tan\theta\tan\beta}$$

$$\tan(\theta-\beta)=\frac{\tan\theta-\tan\beta}{1+\tan\theta\tan\beta}$$



Find the EXACT value of the following using sum and difference formulas:

1) $\sin(15°) = \sin(45-30)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

2) $\tan(75°) = \tan(45+30)$

$= \frac{\tan 45 + \tan 30}{1-\tan 45 \tan 30} = \frac{1+\frac{\sqrt{3}}{3}}{1-1\left(\frac{\sqrt{3}}{3}\right)}$

$= \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right)$

$= \frac{9+6\sqrt{3}+3}{9-3} = \frac{12+6\sqrt{3}}{6}$

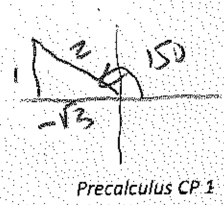
$= 2+\sqrt{3}$

3) $\cos(195°) = \cos(150+45)$

$= \cos 150 \cos 45 - \sin 150 \sin 45$

$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{6}-\sqrt{2}}{4}$



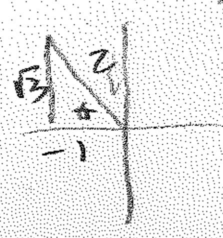
4) $\sin(165°)$

$= \sin(120+45)$

$= \sin 120 \cos 45 + \cos 120 \sin 45$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + -\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$



Section 5.4 – Review of Sum and Difference Formulas

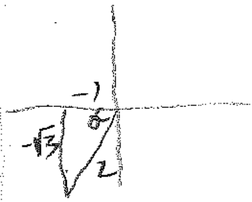
5) $\sin(-75°) = \sin(285)$

$= \sin(240+45)$

$= \sin 240 \cos 45 + \cos 240 \sin 45$

$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{6}-\sqrt{2}}{4}$



6) $\tan(345°) = \tan(315+30)$

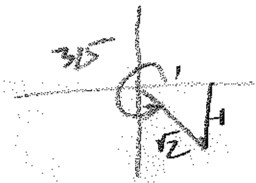
$= \frac{\tan 315 + \tan 30}{1 - \tan 315 \tan 30}$

$= \frac{-1 + \frac{\sqrt{3}}{3}}{1-(-1)(\frac{\sqrt{3}}{3})} = \frac{\frac{-3+\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{\sqrt{3}-3}{\sqrt{3}+3}\left(\frac{\sqrt{3}-3}{\sqrt{3}-3}\right)$

$= \frac{3-6\sqrt{3}+9}{3-9}$

$= \frac{12-6\sqrt{3}}{-6}$

$= -2+\sqrt{3}$



7) $\cos(285°) = \cos(240+45)$

$= \cos 240 \cos 45 - \sin 240 \sin 45$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - -\frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{2}+\sqrt{6}}{4}$

8) $\sin(105°)$

$= \sin(45+60)$

$= \sin 45 \cos 60 + \cos 45 \sin 60$

$= \frac{\sqrt{2}}{2} \frac{1}{2} + \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

Write as sine, cosine, or tangent of an angle. You do NOT have to find the value:

9) $\sin(42°)\cos(17°) - \cos(42°)\sin(17°)$

$= \sin(42-17)$

$= \sin 25°$

10) $\cos 45° \cos 120° - \sin 45° \sin 120°$

$\cos(45+120) = \cos 165°$

Section 5.4 – Review of Sum and Difference Formulas

11) $\dfrac{\tan(19°)+\tan(47°)}{1-\tan(19°)\tan(47°)}$

$\tan(19+47)$

$=\tan(66°)$

12) $\sin 60°\cos 45° - \cos 60°\sin 45°$

$= \sin(60-45) = \sin 15°$

13) $\cos(94°)\cos(18°)+\sin(94°)\sin(18°)$

$=\cos(94-18)$

$=\cos 76°$

14) $\dfrac{\tan 25° + \tan 10°}{1-\tan 25°\tan 10°}$

$=\tan(25+10)$

$=\tan 35°$

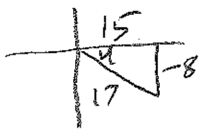
15) $\dfrac{\tan 68° - \tan 115°}{1+\tan 68°\tan 115°}$

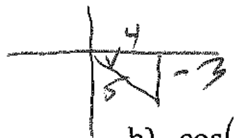
$=\tan(68-115)$

$=\tan(-47°)$

16)Find the EXACT value of the trig function given that

$$\sin u = \frac{-8}{17} \quad \cos v = \frac{4}{5} \text{ and both angles are in Quadrant IV}$$





a) $\tan(u-v)$

$$= \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$= \frac{\frac{-8}{15} - \frac{-3}{4}}{1 + \left(\frac{-8}{15}\right)\left(\frac{-3}{4}\right)} = \frac{\frac{-32+45}{60}}{\frac{60+24}{60}}$$

$$= \frac{13}{84}$$

b) $\cos(u+v)$

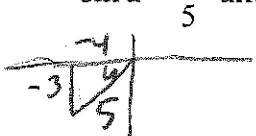
$$= \cos u \cos v - \sin u \sin v$$

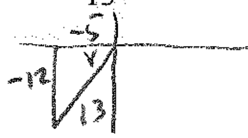
$$= \frac{15}{17} \cdot \frac{4}{5} - \left(\frac{+8}{17}\right)\left(\frac{+3}{5}\right)$$

$$= \frac{60}{85} - \frac{24}{85} = \frac{36}{85}$$

17)Find the **exact value** of the trig function given that

$$\sin u = \frac{-3}{5} \text{ and } \cos v = \frac{-5}{13} \text{ where both u and v are in Quadrant III.}$$





a) $\sin(u-v)$

$$= \sin u \cos v - \cos u \sin v$$

$$= \frac{-3}{5} \cdot \frac{-5}{13} - \frac{-4}{5} \cdot \frac{-12}{13}$$

$$= \frac{15}{65} - \frac{48}{65} = \frac{-33}{65}$$

b) $\cos(u-v)$

$$= \cos u \cos v + \sin u \sin v$$

$$= \frac{-4}{5} \cdot \frac{-5}{13} + \frac{-3}{5} \cdot \frac{-12}{13}$$

$$= \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$$

c) $\tan(u+v)$

$$= \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4} \cdot \frac{12}{5}} = \frac{\frac{15+48}{20}}{\frac{20-36}{20}} = \frac{63}{-16}$$