Section 5.5 – Multiple-Angle Formulas

Don't worry, you do not have to memorize the following formulas, but you have to know how to use them....

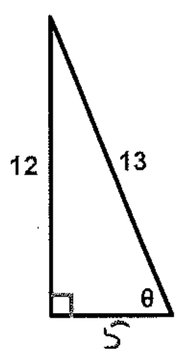
Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Ex. 1) Use the figure to find the exact value of the following:



$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)$
$= \frac{120}{169}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 = \frac{25}{169} - \frac{144}{169} = -\frac{119}{169}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{12}{5}\right)}{1-\left(\frac{12}{5}\right)^2} = \frac{\frac{24}{5}}{\frac{25}{25}-\frac{144}{25}} = \frac{\frac{24}{5}}{\frac{-119}{25}}$

$= \frac{24}{\cancel{5}} \cdot -\frac{\cancel{25}}{119} = -\frac{120}{119}$

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Ex. 2) Find the exact solution of the equation in the interval $[0, 2\pi)$

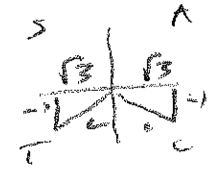
$$\sin 2x + \cos x = 0$$

$$2\sin x \cos x + \cos x = 0$$

$$\cos x(2\sin x + 1) = 0$$

$\cos x = 0$ $\qquad$ $\sin x = -\frac{1}{2}$

$x = \pi/2,\ 3\pi/2$ $\qquad$ $x = \frac{7\pi}{6},\ \frac{11\pi}{6}$



Ex. 3) Find the exact solution of the equation in the interval $[0, 2\pi)$

$$\cos 2x + \sin x = 0$$

$$1 - 2\sin^2 x + \sin x = 0$$

$$-2\sin^2 x + \sin x + 1 = 0$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$\sin x = -\frac{1}{2}$ $\qquad$ $\sin x = 1$

$x = \frac{7\pi}{6},\ \frac{11\pi}{6},\ \frac{\pi}{2}$

Ex. 4) Use a double-angle formula to rewrite the expression.

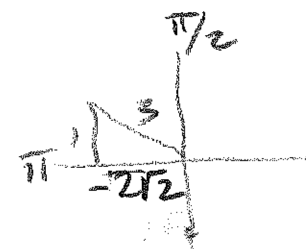
$$6\cos^2 x - 3 = 3(2\cos^2 x - 1) = 3\cos 2x$$

Ex. 5) Find the exact value of the following if $\csc x = 3$, and $\frac{\pi}{2} < x < \pi$

$\csc x = 3$

$\frac{1}{\sin x} = 3$

$\sin x = \frac{1}{3}$



$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$= \frac{2\left(\frac{1}{-2\sqrt{2}}\right)}{1 - \left(\frac{1}{-2\sqrt{2}}\right)^2} = \frac{-\frac{1}{\sqrt{2}}}{1 - \frac{1}{8}}$$

$$= \frac{-\frac{1}{\sqrt{2}}}{\frac{8-1}{8}} = -\frac{1}{\sqrt{2}} \cdot \frac{8}{7}$$

$$= -\frac{8}{7\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{-8\sqrt{2}}{14}$$

$$= -\frac{4\sqrt{2}}{7}$$

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The signs of $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$ depend on the quadrant in which $\frac{\theta}{2}$ lies

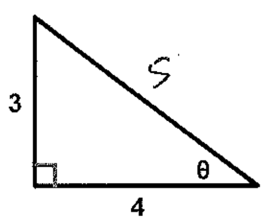
Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

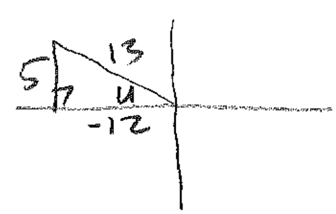
Ex. 6) Use the figure to find the exact value of the following:



$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} = \pm\sqrt{\frac{1-\frac{4}{5}}{2}} = \pm\sqrt{\frac{\frac{1}{5}}{2}} = \pm\sqrt{\frac{1}{10}} = \boxed{\pm\frac{\sqrt{10}}{10}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\frac{4}{5}}{\frac{3}{5}} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{5}\cdot\frac{5}{3} = \boxed{\frac{1}{3}}$$

Ex. 7) Find the exact value of $\tan\frac{u}{2}$ if $\sin u = \frac{5}{13}$ and $\frac{\pi}{2} < u < \pi$.



$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1 - {}^{-12}/_{13}}{5/13} = \frac{25/13}{5/13} = \frac{25}{5} = 5$$

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Ex. 8) Use the half-angle formulas to determine the exact values of:

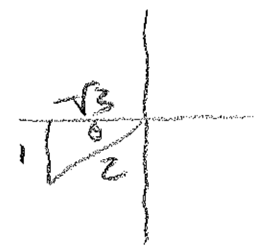
a) $\sin(22.5°)$

$$= \sin\left(\frac{45}{2}\right) = \pm\sqrt{\frac{1-\cos 45}{2}} = \pm\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$

$$= \pm\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \boxed{\pm\sqrt{\frac{2-\sqrt{2}}{4}}}$$

b) $\cos 105°$

$$= \cos\left(\frac{210}{2}\right) = \pm\sqrt{\frac{1+\cos 210}{2}} = \pm\sqrt{\frac{1+\frac{-\sqrt{3}}{2}}{2}}$$



$$\boxed{= \pm\sqrt{\frac{2-\sqrt{3}}{4}}}$$

Ex. 9) Use the half-angle formulas to simplify the expression.

$$\frac{1-\cos 8x}{\sin 8x} = \tan\frac{8x}{2} = \tan 4x$$

HW: 5.5 p.415: 3, 5, 9, 13, 19, 21, 24, 27, 35, 37, 41, 42, 50, 53, 55