

Section 5.5 – Multiple-Angle Formulas

Don't worry, you do not have to memorize the following formulas, but you have to know how to use them....

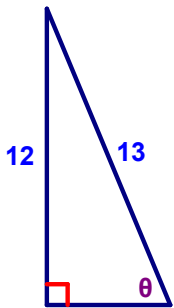
Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Ex. 1) Use the figure to find the exact value of the following:



$$\sin 2\theta =$$

$$\cos 2\theta =$$

$$\tan 2\theta =$$

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Ex. 2) Find the exact solution of the equation in the interval $[0, 2\pi)$

$$\sin 2x + \cos x = 0$$

Ex. 3) Find the exact solution of the equation in the interval $[0, 2\pi)$

$$\cos 2x + \sin x = 0$$

Ex. 4) Use a double-angle formula to rewrite the expression.

$$6\cos^2 x - 3$$

Ex. 5) Find the exact value of the following if $\csc x = 3$, and $\frac{\pi}{2} < x < \pi$

$$\tan 2x =$$

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****The signs of $\sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2}$ depend on the quadrant in which $\frac{\theta}{2}$ lies****

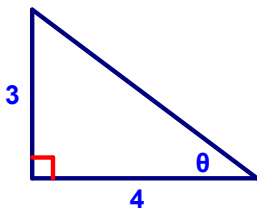
Half-Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$

Ex. 6) Use the figure to find the exact value of the following:



$$\sin \frac{\theta}{2} =$$

$$\tan \frac{\theta}{2} =$$

Ex. 7) Find the exact value of $\tan \frac{u}{2}$ if $\sin u = \frac{5}{13}$ and $\frac{\pi}{2} < u < \pi$.

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Ex. 8) Use the half-angle formulas to determine the exact values of:

a) $\sin(22.5^\circ)$

b) $\cos 105^\circ$

Ex. 9) Use the half-angle formulas to simplify the expression.

$$\frac{1 - \cos 8x}{\sin 8x}$$

HW: 5.5 p.415: 3, 5, 9, 13, 19, 21, 27, 35, 37, 41, 53, 55